ACI News Letter

Vol. 23 No. 5 JOURNAL of the AMERICAN CONCRETE INSTITUTE January 1952

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ACI
News Letter Contents

January 1952

On the Cover—Tyler Davidson fountain superimposed on Carew Tower, Cincinnati, Ohio. The Netherland Plaza Hotel, part of the same building group, may be seen in the background.

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1952 Convention Program

Emphasis at the 1952 convention, Netherland Plaza Hotel, Cincinnati, February 26-28, will be on economy in design and construction and on good concreting practice to produce more durable structures. Details of the papers to be presented begin on p. 4.

Of two papers on the design and construction the first will report economic comparisons for 260 different designs for conventional and prestressed concrete reservoirs. The second will review the design of a prestressed concrete bridge and the construction problems in assembling and prestressing the precast units.

Durability of concrete will be considered in five papers. The important field of concrete railroad structures will be reviewed by representatives of two major railways. Durability in pavements and hydraulic structures will complete this part session. A round-table discussion on curing will point up the requirements for pavements, structural concrete, mass concrete and canal linings. Fundamental principles of curing and the action of various curing agents will be discussed.

An introduction to the session on ultimate load design will give the historical background of the ultimate load theories while other papers will summarize research programs in this field, demonstrate the practical application to the design of sections and discuss load factors.

Under materials and properties the thermal expansion of aggregates, effects of temperature changes on concrete as influenced by aggregates, calcium chloride in concrete, and the use of fly ash to reduce cement-aggregate expansion will be discussed.

In the design session, field practice in lightweight concrete, economical design of concrete beams, developments in reinforcing steel and concepts of safety, and the analysis of skewed rigid frames will be reported.

Papers on applications of vacuum concrete, temperature control in mass concrete, small building construction and insulation of forms to protect new concrete from freezing will comprise the session on construction. A movie will show a machine method of lining pipes with concrete.

The annual research session will provide glimpses of progress in various studies of concrete underway at public and private laboratories.

The panel discussion will emphasize the problems on the small concrete job. Panel members are being selected on the basis of their experience on this type of work.

Continued on p. 4

Program preview, 48th Annual ACI Convention, Netherland Plaza Hotel Cincinnati

February 26-28, 1952

TUESDAY, FEBRUARY 26

9 a.m.—Fourth Floor Foyer—Convention registration begins—Members and nonmembers—Registration fee $2.00

Technical committee meetings as scheduled

10:30 a.m.—Pavilion Caprice—Convention called to order by President Harry F. Thomson

PRESTRESSED CONCRETE

Relative Economy of Conventional and Prestressed Reinforced Concrete Reservoirs—George C. Ernst, C. O. Brunken and A. R. Riveland, University of Nebraska, Lincoln

Cylindrical wall designs resulting from the use of various stress combinations for conventional and prestressed conditions are compared, and the most advantageous are selected for making an economic study. Quantities of concrete and steel resulting from designs for 180 complete reservoirs and 80 additional designs reveal that those of prestressed concrete would have a lower first cost than the conventional designs if unit cost of the prestressed wall in place does not exceed approximately twice that for the conventional wall. In most cases of unrestricted site location with the same controlling water elevation, reservoirs above ground were more economical than underground reservoirs insofar as first cost is concerned. However, underground reservoirs could compete with those above ground for capacities greater than 2.5 million gallons. As to materials, prestressed construction saves from 40 to 60 percent for steel and from 0 to 35 percent for concrete.

Field Problems in Constructing a Prestressed Concrete Bridge—Clair L. Johnson, Johnson and Anderson, Pontiac, Mich.

Discusses design considerations for a prestressed concrete bridge in which the girders were assembled from precast blocks grouted after completion of the members. Field experience showed that a reduction in the number of shapes of blocks used was desirable and resulted in lower over-all cost. Breakage of blocks and difficulties in grouting between blocks were overcome by modification of assembling technique, changes in design and the development of re-usable joint covers. Bid prices on three successive jobs indicate that with familiarity with prestressed construction the cost tends to drop.

2:00 p.m.—Pavilion Caprice—Vice-President A. T. Goldbeck, chairman

DURABILITY

Field Studies of Durability of Railroad Concrete—R. W. Gilmore, B & O Railroad, Cincinnati, Ohio

Report on conditions affecting durability of concrete structures, and discussion of design modifications and changes in construction and maintenance procedures to prolong their useful life.

General Condition of Concrete Railroad Structures and Research Program on Surface Waterproofers—C. B. Porter, Chesapeake and Ohio Railway, Richmond, Va.

Survey of structures of various ages and an evaluation of factors affecting their

durability. Brief report of research on testing and developing waterproofing materials.

Concrete Pavements Can be Durable—Frank H. Jackson, Bureau of Public Roads, Washington, D. C.

A brief discussion of the factors that control the durability of concrete pavements. "Structural performance" and "durability" are defined. It is shown that although strength alone will satisfy the usual structural design requirements, durability is also of great importance particularly in the northern states. The role of air entrainment in improving the durability of concrete pavements, particularly under the attack of chloride salts used for ice removal, is discussed and present thinking on this subject summarized. It is concluded that, although air entrainment will greatly increase resistance to straight freezing and thawing, it is not, as presently applied in practice, the final answer to the problem of salt scaling, particularly where large quantities of raw salt are used to melt ice quickly.

Concrete Can be Durable in Hydraulic Structures—L. H. Tuthill, Bureau of Reclamation, Denver, Colo.

The necessity of controlling volume change and providing impermeability in mass concrete hydraulic structures contribute to problems of durability. Mix design, placement methods, use of admixtures and other measures to increase the serviceability of such work are considered.

Durability of Hydraulic Structures—Byram W. Steele, consulting engineer, Miami, Fla.

Durability of concrete is a relative term which must be evaluated in terms of the overall conditions applicable to the structure under consideration. "That concrete is good enough" is a statement often heard in discussing certain structures. In many cases the concrete *is* good enough; to have spent more time and money making better concrete would have been an economic waste.

In hydraulic structures, however, the concrete is too often *not good enough*. Within a few years many such structures show serious signs of deterioration because water finds its way into and through the concrete in many places. This phase of concrete durability is discussed and "impermeability" is stressed as the dominant factor in long-time durability in concrete.

CURING

Round-table discussion—Sponsored by Committee 612, Mark Morris, chairman

Current Concepts of Fundamental Principles of Curing—A. G. Timms, National Bureau of Standards, Washington, D. C.

Brief review of present thinking on concrete curing; what is desired to be accomplished and general methods of how it is done.

Current Practices in Curing Concrete Pavements—D. L. Robinson, Missouri State Highway Dept., Jefferson City

Discussion of the methods of curing concrete permitted under the specifications of State highway departments. Saturated coverings of cotton felt or jute mats or two thicknesses of burlap are almost universally permitted. Other widely permitted types, in order, are covering with waterproof paper, saturated earth or ponding, sealing with membranes and covering with hay or straw. Length of curing period required is found to vary considerably except that 72 hours is the minimum permitted for normal cement.

Recommended Practices for Curing Structural Concrete—H. J. Gilkey, Iowa State College, Ames

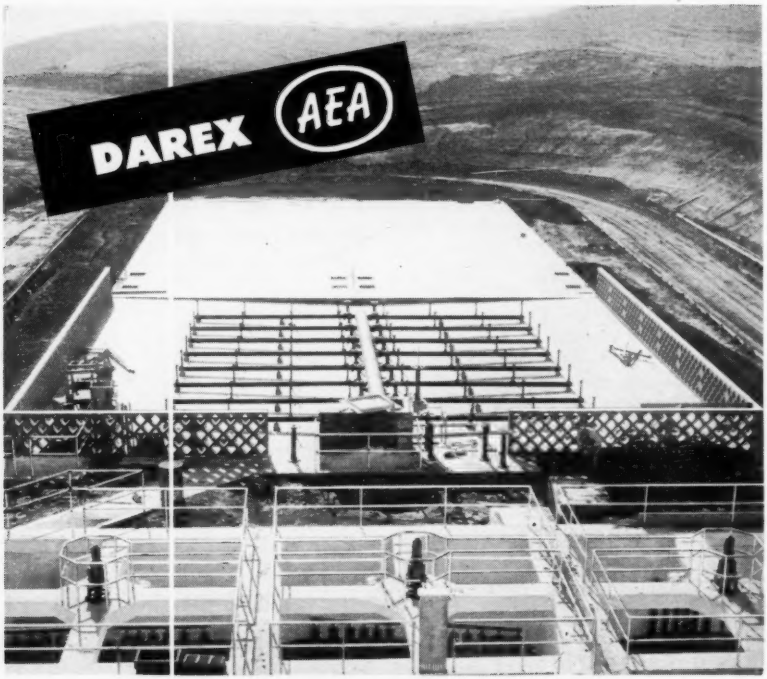
Summarizes the factors in curing structural concrete. Points out where and how these differ from those encountered in other types of concrete. Discusses current practices, especially form removal and temperature and moisture control in cold weather.

Recommended Practices for Curing Mass Concrete—W. R. Johnson and Roy W. Carlson, Bureau of Reclamation, Denver, Colo.

Discusses water and membrane curing of mass concrete. Gives data on protection afforded by tarpaulins and wood and steel forms to concrete placed at low temperatures.

Continued on p. 7

Another job better done with



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A special "Harvey Whipple Luncheon" in recognition of more than 32 years of service as ACI Secretary will be held Thursday noon. Details will be announced at registration time or before. Plan to attend.

Continued from p. 5

Recommended Practice for Curing Canal Linings—G. E. Burnett, Bureau of Reclamation, Denver, Colo.

Present practice is to cure concrete canal linings with liquid sealing compounds; white-pigmented compound being preferred by the Bureau of Reclamation. Good material, proper time of application, adequate film thickness and uniform coverage are important factors in effective and economical curing by this method.

Curing Methods for Concrete—H. C. Vollmer, Emergency Transportation Dept., Washington, D. C.

The more successful of available curing agents are listed and described; their advantages and disadvantages, and reasons for rise in usage under various circumstances are discussed.

Electric Curing of Concrete—Chuzo Itakura, Hokkaido University, Sapporo, Japan

Describes an electrical method of curing concrete in cold weather which has been used in about 80 structures in northern Japan. The structures ranged from simple piers for wooden buildings to abutments and piers 80 ft high. The system, employing a network of electrodes, is applicable to plain or reinforced concrete. The cost of curing by this method varies from 7 to 10 percent for plain concrete to 10 to 15 percent for reinforced concrete. Effectiveness and efficiency would be improved by reliable automatic devices for controlling the current.

WEDNESDAY, FEBRUARY 27

Concurrent sessions will be held morning and afternoon in the **Pavilion Caprice** and **Parlors A-D**

9 a.m.—Pavilion Caprice—President Harry F. Thomson, chairman

ULTIMATE LOAD DESIGN

Introduction—Leo H. Corning, Portland Cement Assn., Chicago, Ill., session leader

Review of the historical background—from Talbot to present day. Discussion of codes in countries permitting design by ultimate load theories.

Why Design by Ultimate Load Theories?—Boyd G. Anderson, Ammann and Whitney, New York, N. Y.

Discusses following reasons for ultimate design: to bring the design of concrete members into one common rational basis; to make the factor of safety for all shapes the same; to rationalize the use of load factors so that different factors of safety can be assigned to different types of loading and different types of structures; to prevent uneconomical use of compressive reinforcement; to simplify design procedure; to better predict performance of structures subject to long-duration impulsive loads; and to determine capacities of prestressed concrete members.

Fundamental Concepts in Ultimate Load Design of Reinforced Concrete Members—Eivind Hognestad, University of Illinois, Urbana

Traces development of design methods for reinforced concrete structural members through the three stages of an engineering science; empirical, scientific and mature. Discusses various ultimate design equations in relation to the three prevailing philosophies regarding mode of failure: same safety against all modes of failure, higher safety against less desirable modes of failure, and prohibiting undesirable modes of failure.

Research on Ultimate Strength of Reinforced Concrete Members—C. P. Siess and Eivind Hognestad, University of Illinois, Urbana

Concerned with tests to determine the manner of failure and ultimate strength of reinforced concrete structural members. Emphasis will be placed on tests made in the United States, but foreign tests also will be mentioned.

Available test data will be reviewed critically, their scopes will be defined, their limitations or deficiencies will be pointed out, and at least by inference, the additional test data desired or required will be mentioned.

Design of Sections by Ultimate Load Theories—Raymond C. Reese, Toledo, Ohio

After a few comments on the fundamental philosophy of ultimate load design, a grouping is made of those formulas which are needed for practical design (rectangular stress prism). Designs are carried out for slabs and beams and for columns with and without bending by the conventional and ultimate load design methods. Some comparisons are made of the results as regards amount of space occupied, time-sagging of members and cost of construction. Remarks are offered on the appropriateness of using elastic frame analysis in plastic theories, also discussion of the various ultimate stress prisms.

Load Factors in Ultimate Design—T. Y. Lin, University of California, Berkeley

Compares load factors in ultimate design and allowable stresses in conventional design. Considers the basis for the determination of load factors and makes tentative recommendations for their selection.

9 a.m.—Parlors A-D—Past President Frank H. Jackson, chairman

MATERIALS AND PROPERTIES

Thermal Expansion of Aggregates and Concrete Durability—Edwin J. Callan, Waterways Experiment Station, Jackson, Miss.

Describes methods developed to obtain simply the thermal coefficients of fine and coarse aggregates. Concretes made of various aggregate combinations were tested in accelerated freezing and thawing, yielding durability factors, *DFE*, for each combination. Statistical analysis of the *DFEs* showed that durability of these concretes was reduced when the differential expansion of mortar and coarse aggregate increased. Stresses set up by differential expansion and effects on concrete durability are discussed briefly. Thermal effects should be considered in choosing aggregates for durable concrete.

Effects of Temperature Changes on Concrete as Influenced by Aggregates—Delmar Bloem, Stanton Walker and W. G. Mullen, National Sand and Gravel Assn., Washington, D. C.

Summarizes results of tests of concrete and mortar subjected to water and air ranging from 40-140 F with a varying rate of change in temperature. Changes in length, weight, dynamic modulus and flexural strength were measured on specimens containing several different fine and coarse aggregates having thermal coefficients of expansion ranging from relatively low to relatively high. Principal findings of the tests were: thermal coefficients of expansion of concrete and mortar containing different aggregates varied approximately in proportion to the thermal coefficient and quantity of aggregate in the mixture; an approximation of the thermal coefficient of expansion of aggregate may be made from determinations of the thermal coefficients of concrete of varying proportions; changes in temperature were destructive of the concrete with sudden changes being much more severe than slower ones; and concretes having higher coefficients of expansion were less resistant to temperature changes than concrete with lower coefficients, no relationship was found between resistance of concrete to temperature changes and differences between thermal coefficients of aggregates and mortar.

Calcium Chloride in Concrete—J. J. Shideler, Bureau of Reclamation, Denver, Colo.

Gives results of studies of the effects of varying quantities of calcium chloride on the qualities of the resultant concrete.

Fly Ash to Reduce Expansion of Cement-Aggregate Combinations—C. H. Scholer and Gerald K. Smith, Kansas State College, Manhattan, Kans.

Describes laboratory tests of beam specimens, in which 20 to 30 percent of the cement was replaced with fly ash, subjected to two different accelerated exposures which showed excellent correlation with field service records. The fly-ash effectively inhibited cement-aggregate reaction in the concrete while imparting better workability and finishing properties.

Continued on p. 10

The good performance of Masterbuilt iron armored concrete floors in two other Hinde & Dauch plants resulted in the company's decision to install this modern industrial floor in the fine new plant shown above, located in Richmond, Va.

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Continued from p. 8

LUNCHEON

12:15 p.m.—Hall of Mirrors—President Thomson presents ACI awards

Report of Annual Election and Introduction of New Officers

Recognition of Retiring Directors and Officers

Retiring President's Address

2:30 p.m. — Pavilion Caprice — Vice-President Henry L. Kennedy, chairman

DESIGN

Field Practice in Lightweight Concrete—J. A. Murlin, George L. Dahl, Architects and Engineers, and Cedric Willson, Texas Industries, Dallas, Texas

Brief comparison between concrete made with expanded shale and clay aggregates and that made with other lightweight materials and the usual heavier materials. Covers the design of lightweight structural concrete members and their economy as compared to heavy concrete taking into account the cost differential between the two materials. A practical and simple method for the design and control of lightweight structural concrete, both ready- and job-mixed, which has worked well in the field will be discussed. Other sections will deal with methods and comparative costs of mixing, placing and finishing; use of admixtures; and the economy of using expanded clay or shale structural fill concrete as compared with other lightweight materials which are being specified for this purpose.

Economy of Concrete Beams—Herbert A. Sawyer, Jr., University of Connecticut, Storrs

Relationships and charts are presented from which may be readily determined the cross-sectional dimensions for maximum economy of materials of a beam or slab with given span and loading. Whitney's theory is used to express beam strength. The steel saving possibilities of economical dimensioning are considered and found especially important.

These optimum dimensions almost never correspond to balanced design, but depend on relative material costs, relative material strengths, relative length of beam reinforced for diagonal tension and relative importance of beam dead weight.

Optimum dimensions for the Whitney theory also serve as a general guide for efficient design by the elastic theory.

Developments in Reinforcing Steel and Concepts of Safety—F. Hajnal-Konyi, London, England

Reports results of testing 38 beams to determine the efficiency of 17 different types of reinforcing bars. In addition to observations of deflection and crack formation, strain measurements were taken on 11 beams which confirm the straight line distribution of the average strain, *i.e.*, the strain measured over cracks in the tensile zone.

The tests demonstrated that working stresses on which present design is based do not give a satisfactory measure of safety. Calculation of the ultimate moments by Whitney's method based on the actual strength of the materials agrees very well with the test results.

In assessing test results three types of failure must be considered; sudden (or brittle), without warning; yielding, with a sudden increase of deformation on reaching the maximum load which can be maintained until the deformation is substantial; and gradual (or tough) with ample warning of the approaching danger, allowing an increase of load even at a stage where failure appears imminent.

Analysis of Skewed Rigid Frames—James P. Michalos, Iowa State College, Ames

A numerical procedure for the analysis of single-span skewed rigid frames and arches subjected to loads or deformations in any direction. The structure may have any shape and any variation in cross section along its length. Several examples are included.

2:30 p.m.—Parlors A-D—Past President Robert F. Blanks, chairman

CONSTRUCTION

Applications of Vacuum Concrete—K. P. Billner, Vacuum Concrete Corp., Philadelphia, Pa.

Vacuum concrete processes were introduced in this country a number of years ago and their applications in building construction have been reported in the ACI JOURNAL. During the last few years these processes have become known and adopted in many foreign countries, particularly in Europe, and this paper deals with these foreign developments, singling out new ways and methods in construction.

Concrete Temperature Control at Pine Flat Dam—S. D. Burks, Sacramento District, Corps of Engineers, Sacramento, Calif.

Preconstruction studies conducted for Pine Flat Dam—a concrete gravity structure 440 ft high and containing 2,250,000 cu yd of concrete—indicated that the tendency for cracks to occur from temperature-induced volume changes could be minimized by the adoption of certain control measures. This paper briefly relates the studies made to predict temperature behavior, lists the broad requirements of the control program adopted, describes the methods adopted by the contractor in response to such requirements, and relates the on-the-job temperature experiences which attest to the success of the program for the first million cu yd of concrete placed in the dam.

Small Building Construction—C. O. Christenson, National Assn. of Home Builders, Washington, D. C.

Describes application of lightweight aggregates, manufacture and use of prefabricated building elements, and the control of concrete quality in small building work where the services of a concrete expert usually are not available.

Insulation for Protection of New Concrete in Winter—L. H. Tuthill, R. E. Glover, C. H. Spencer and W. B. Bierce, Bureau of Reclamation, Denver, Colo. (See November JOURNAL for complete paper)

Recent investigations show that new concrete with 1 percent calcium chloride, if kept from dropping below 50 F for 3 days, has sufficient protection from freezing. As an extra precaution, 3 more days protection from dropping below 32 F may be required. Since it is during the first 3 days that considerable heat is generated by the setting cement, it has been learned that insulation, within practicable limits, is capable of retaining enough of this heat to keep new concrete from dropping below these temperatures. This avoids construction of special enclosures, fuel costs, and danger of fire. Examples of field applications and tests of insulation are reported together with resultant concrete temperatures.

Centriline Process of Pipe Lining—a movie prepared by Raymond Concrete Pile Co., New York, N. Y.

Describes preparation of pipe for lining, installation and arrangement of lining machinery, and the application of the mortar lining to the interior of the pipe.

THURSDAY, FEBRUARY 28

9:00 a.m.—Pavilion Caprice—Research—An open session of Committee 115, Research—S. J. Chamberlin, chairman; George W. Washa, secretary

A group of short papers on research techniques and preliminary results on fatigue test of concrete beams, weathering and freezing and thawing tests, impulsive loading, stress distribution, volumetric changes and other subjects. Papers not for publication or press release.

12:15 p.m.—Harvey Whipple Luncheon

2 p.m.—Pavilion Caprice—Panel discussion—with emphasis on concrete problems on the small job. Past President H. J. Gilkey, presiding

Final selection of panel members still to be made but will consist of those who can speak from first-hand experience on overcoming difficulties in the field.

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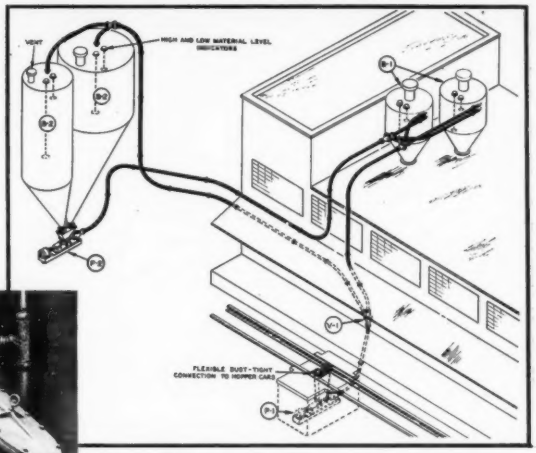

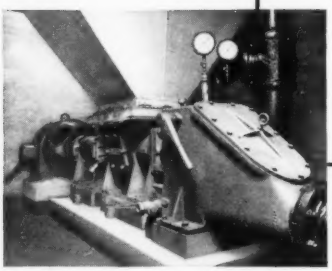

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ACI Plans Participation in U.S. Centennial of Engineering

The American Concrete Institute is planning to make its 1952 fall regional meeting a cooperative effort with the convocation of engineers to be held in Chicago, September 3-13, 1952, as a part of the U. S. Centennial of Engineering.

Arrangements for a technical program under the joint sponsorship of ACI and ASCE for sessions on September 11 and 12 are being handled by a technical program committee consisting of C. E. Wuerpel, chairman, A. E. Cummings, L. H. Corning, M. J. Holley, Jr., H. F. Thomson, ex-officio.

Arrangements for other than the technical activities are being handled by a committee under the chairmanship of H. C. Delzell.

As representatives of ACI, Messrs. Delzell and Wuerpel are members of the Centennial of Engineering General Arrangements Coordinating Committee and Technical Program Coordinating Committee, respectively.

Subsequent issues of the JOURNAL will carry more detailed information regarding both the general arrangements and the technical program. It is not too early to plan to attend the "Centennial" which will be one of the outstanding engineering convocations of the century. At the latest report, 36 national and international engineering societies had indicated an intention to participate.

Proposed Changes in ACI Bylaws

Notice is hereby given to the membership of the American Concrete Institute that at the 48th Annual Convention, Cincinnati, Ohio, February 26-28, 1952, consideration will be given to a petition for Bylaws amendments.

Changes proposed in Article I clarify the definitions of membership grades and outline in more detail the method of electing Honorary Members.

Changes proposed in Article II list officers and define the Board of Direction. They also remove the Secretary-Treasurer from the Board of Direction and eliminate the requirement that he be appointed annually.

Other changes proposed are editorial.

A Petition

The undersigned members of the American Concrete Institute hereby petition the Board of Direction to submit to the membership for adoption, in accordance with Article VI of the Bylaws, the following proposed changes in the Bylaws (the left-hand column repeats the present wording of the Bylaws; the right-hand column the proposed rewording or change):

ARTICLE I—MEMBERS

PRESENT

Section 1. This Institute shall consist of Members, Corporation Members, Contributing Members, Junior Members, Student Members and Honorary Members interested in furthering the Institute's objects as set forth in its Charter.

Sec. 2. A Member shall be an individual.

A Corporation Member shall be a firm, corporation, society, agency of government or other organization. A Corporation Member shall name one individual as its representative who will enjoy all membership rights and privileges.

A Contributing Member shall be an individual, firm, corporation, society, agency of government or other organization wishing to give larger support to Institute activities through the payment of larger dues. Contributing Members, other than individuals, shall name representatives as do Corporation Members.

A Junior Member shall be a person less than 28 years old.

A Student Member shall be less than 28 years old and a student in residence at a recognized technical or engineering school.

An Honorary Member shall be a person of eminence in the field of the Institute's interest, or one who has performed extraordinarily meritorious service to the Institute.

Sec. 3. All classes of Members, except Honorary Members and Student Members, shall be proposed by at least one Member of the Institute and elected by a two-thirds vote of the Board of Direction. An Honorary Member shall be elected by unanimous vote of the Board of Direction. A Student Member shall be proposed by a member of his school's faculty, who need not be a member of the Institute, and elected by a two-thirds vote of the Board of Direction.

Sec. 4. All members shall have all rights and privileges of membership as determined by the Board of Direction except that a Junior or Student Member shall not vote nor hold office. The status of a Student Member shall change automatically to that of Junior Member on the first anniversary of his membership succeeding the date on which he ceases to be a student in residence. The status of a Junior Member shall be changed to that of Member on the first anniversary of his membership after he becomes 28 years of age.

Sec. 5. Applications for and resignations from membership and requests for change of representatives of Corporation or Contributing Memberships shall be

PROPOSED

Section 1. This Institute shall consist of Honorary Members, Corporation Members, Contributing Members, Members, Junior Members and Student Members.

Sec. 2. An Honorary Member shall be a person of eminence in the field of the Institute's interest, or one who has performed extraordinarily meritorious service to the Institute. He shall have the same rights and privileges as a Member, but shall not be subject to dues.

The total number of Honorary Members of the Institute and the maximum number to be elected in any one year shall be at the discretion of the Board of Direction.

The Committee on Honorary Membership shall consist of the three Past President members of the Board of Direction. The chairmanship of this committee shall be vested each year in that person beginning his third-year term as a Past President member of the Board. This group shall serve as a screening committee for the formulation of recommendations to the Board. If one or more members of the regularly constituted committee on Honorary Membership is deceased or inactive, the President shall appoint to each vacancy some regular member of the Board of Direction to serve through the period of shortage, the Senior Past President non-appointed member of the committee functioning as its chairman.

The Committee on Honorary Membership is expected to be continuously alert for deserving prospects and open to considered suggestions from all sources. It shall submit a formal report to the President of the Institute each year at least thirty days prior to the Fall meeting of the Board. The annual report is mandatory even though there be no recommendations.

Sec. 3. A Member shall be an individual.

A Corporation Member shall be a firm, corporation, society, agency of government or other organization.

A Contributing Member shall be an individual, firm, corporation, society, agency of government or other organization electing to give greater support to Institute activities through the payment of larger dues. Any Contributing or Corporation Member, other than an individual, may name a personal representative who shall enjoy all membership rights and privileges.

A Junior Member shall be a person less than 28 years of age.

A Student Member shall be less than 28 years of age and a registered student at a recognized technical or engineering school.

Sec. 4. All classes of Members, except Honorary Members and Student Members, shall be sponsored by at least one Member of the Institute. An Honorary Member shall be elected by unanimous vote of the Board of Direction. A Student Member shall be sponsored either by a Member of the Institute or by a member of his school's faculty, who need not be a Member of the Institute.

Sec. 5. All Members shall have all rights and privileges of membership as determined by the Board of Direction except that a Junior or Student Member

presented in writing to the Secretary-Treasurer. Resignations may be accepted only from members whose dues are not more than 60 days in arrears, except by special action of the Board of Direction.

shall neither vote nor hold office. The status of a Student Member shall change automatically to that of Junior Member on the first anniversary of his membership succeeding the date on which he ceases to be a registered student. The status of a Junior Member shall be changed to that of Member on the first anniversary of his membership after he becomes 28 years of age.

Sec. 6. Applications for and resignations from membership and requests for change of representatives of Corporation or Contributing Members shall be presented in writing to the Secretary-Treasurer. Resignations may be accepted only from Members whose dues are not more than 60 days in arrears, except by special action of the Board of Direction.

ARTICLE II—OFFICERS

Section 1. The officers shall be a President, two Vice-Presidents, twelve Directors, and the Secretary-Treasurer, who with the three latest, living Past Presidents who continue to be Members, shall constitute the Board of Direction.

Section 1. The officers shall be a President, two Vice-Presidents, twelve Directors, the three latest living Past Presidents who continue to be Members of the Institute, and the Secretary-Treasurer. The President, the Vice-Presidents and the Directors shall be elected from the Institute membership. The Secretary-Treasurer shall be appointed by the Board of Direction.

Sec. 2. The President, Vice-Presidents, Directors, and five members of a Committee on Nominations shall be elected by letter ballot of the Institute membership. The Secretary-Treasurer shall be appointed annually by the Board of Direction.

Sec. 2. The President, Vice-Presidents, Directors and the three latest living Past Presidents who continue to be Members, shall constitute the Board of Direction.

Sec. 3. Before September 1 of each year the Committee on Nominations shall report to the Secretary-Treasurer of the Institute the candidates nominated for offices to become vacant at the next annual convention and twenty candidates for membership on the Committee on Nominations which is to serve in the following year. In the selection of candidates for Directors, the Committee on Nominations shall have due regard for diversity of professional and geographical representation. Each candidate for Board membership must have given consent to his nomination before the report is published. The Secretary-Treasurer shall cause notice of all such nominations to be transmitted to the membership of the Institute at least 120 days prior to the next ensuing annual convention. Upon petition to the Board of Direction signed by at least ten members of the Institute, additional nominations for offices or for membership on the Committee on Nominations may be made within 30 days thereafter.

The complete list of nominations shall be submitted 60 days before the next convention to the Institute membership for letter ballot to be canvassed at 5 p.m. on the first day of the convention and the result announced at a session of the convention on the second day. The candidate for any office receiving the most votes shall be declared elected and the candidate receiving the most votes for membership on the Committee on Nominations shall be chairman of that committee; the four next highest shall be declared elected members of the committee. With these five the three past-president members of the Board of Direction shall serve, making a total membership of eight.

Should any member of the Committee on Nominations thus chosen fail, within fifteen days of formal notice from the Secretary-Treasurer, to make written

Sec. 3. Change last sentence paragraph 1 to read as follows, "By petition to the Board of Direction signed by at least ten Members of the Institute, within 30 days thereafter, additional nominations for offices or for membership on the Committee on Nominations may be made."

First sentence paragraph 2 insert "annual" before "convention."

acceptance of service, a vacancy shall occur to be filled by the candidate receiving the next greatest number of votes and so on until the five elected places on the committee shall be filled.

Sec. 4. Terms of office shall be as follows: President, one year; Secretary-Treasurer, one year; Vice-President, two years, with one Vice-President elected alternate years; Directors, three years, with four Directors elected each year. A year is here construed as the period between the reports of tellers on canvass of ballots for Board members at two successive annual conventions.

Sec. 4. Line 3 insert "in" to read "elected *in* alternate" etc.

Sec. 5. The President shall be ineligible for more than one re-election to the same office until the lapse of at least one term. Vice-Presidents and Directors shall be ineligible for re-election to the same office until the lapse of at least one year.

Sec. 5. Delete "more than one" from first line.

* * * * * *

Sec. 9. In the event of disability or neglect in the performance of his duty of any officer of the Institute, the Board of Direction shall declare the office vacant.

Sec. 9. In the event of disability of any officer of the Institute, or neglect in the performance of the duties of the office, the Board of Direction shall declare the office vacant.

Sec. 10. The Board of Direction shall have general supervision of the affairs of the Institute. At a meeting held in the week of the annual convention it shall appoint a Secretary-Treasurer for a term of one year. It shall authorize and appoint the chairmen of such administrative and technical committees and assign to them such duties and such authority as it deems needful to carry on the work of the Institute. Additional committee members shall be appointed by the President.

Sec. 10. Delete second sentence.

Sec. 11. There shall be an Executive Committee of the Board of Direction consisting of the President; Secretary-Treasurer, and three of its members appointed by the Board of Direction.

Sec. 11. Delete "Secretary-Treasurer.'

ARTICLE III—MEETINGS

* * * * * *

Section 2. The Board of Direction shall meet at least twice each year, the time and place fixed by the Board.

Sec. 2. Insert "at" to read "each year *at* the" etc.

ARTICLE VI—AMENDMENTS

Section 1. Proposed amendments to these Bylaws, signed by at least fifteen Members, if presented in writing to the Board of Direction ninety days before the annual convention, shall be mailed to the membership at least thirty days prior to the annual convention. These amendments may be discussed and amended at the annual convention and be passed to letter ballot by a two-thirds vote of those present. Two-thirds of the votes cast by letter ballot canvassed within ninety days thereafter shall be necessary for their adoption.

Section 1. Last sentence, substitute "after mailing ballot forms" for "thereafter."

Signed by:

Robert F. Blanks
A. E. Cummings
Harmer E. Davis
Herbert J. Gilkey
A. T. Goldbeck
Frank H. Jackson
Henry L. Kennedy
Frank Kerekes
G. L. Lindsay
I. E. Morris
Nathan M. Newmark
Walter H. Price
R. W. Spencer
I. L. Tyler
Harvey Whipple
Charles E. Wuerpel

MEMORIAL*

FRANK ERWIN RICHART

Frank Erwin Richart, who died on July 16, 1951, contributed greatly to the establishment of the American Concrete Institute as a recognized source of authoritative information on concrete in all its phases and for all of its uses. He served the Institute in many ways and in many capacities including that of President. He leaves behind him a host of friends and an enviable record of professional accomplishment.

Frank Richart was born in Lena, Illinois, on May 11, 1892. After the usual preparatory work he entered the Engineering College of the University of Illinois, from which he received the degree of Bachelor of Science in 1914, that of Master of Science in 1915 and the Degree of Civil Engineer in 1922. He joined the faculty of his alma mater in 1916 in the Department of Theoretical and Applied Mechanics where he continued until his death, since 1931 having the title Research Professor of Engineering Materials. He early became recognized as one of the outstanding writers on structural materials, particularly in the field of concrete and reinforced concrete. He was an author of many technical papers published by the University and by national engineering organizations.

No brief statement can do justice to Frank Richart's achievements nor to the deep friendships which he inspired. His numerous contributions to the work of the American Concrete Institute represented only one of the many outlets for his abilities.

He became a member of the American Concrete Institute in 1917 and his first recorded formal contribution to the work of the Institute was made in 1921 when he presented a paper dealing with the tests of a flat slab floor. He served for many years on numerous technical committees and most of the several administrative committees of the Institute. He was a member of the Board of Direction from 1934 to 1936, Vice-President in 1937 and 1938 and President in 1939. He was Wason Medalist in 1938, and in 1950 he was the first recipient of the Alfred E. Lindau Award for "outstanding contributions to reinforced concrete design practice." He was elected honorary member in 1951.

In addition to his membership in the American Concrete Institute he was active in a number of other technical organizations. He served on the Executive Committee, on the Board of Directors and as Vice-President of the American Society for Testing Materials, declining nomination for the Presidency in 1951 because of ill health. He was an honorary member of that Society. He was prominent in its Committee work and served as Chairman of its Committee C-9 on Concrete and Concrete Aggregates. He was active in the American Society of Civil Engineers and was one of its representatives on the Joint Committee on Standard Specifications for Concrete and Reinforced Concrete. He was Illinois Faculty representative on the Intercollegiate Conference (Big Ten), a Director of the University of Illinois Athletic Association, member of Tau Beta Pi, Sigma Xi, Phi Kappa Phi and of many other engineering organizations.

While the American Concrete Institute knew Frank Richart as a warm personality and close friend who had contributed much to the technology of concrete and concrete aggregates, it was not unaware of his high attainments in his chosen field of education. He was a teacher of great ability as can be attested by numerous engineers throughout the world who received training under him.

BE IT THEREFORE RESOLVED: That this memorial to the life and services of Frank Erwin Richart be sent to his family with an expression of the sincere sympathy of the members of the American Concrete Institute; that it be sent to the President of the American Concrete Institute; and that it be duly spread upon the Minutes of the Institute's Board of Direction.

*Adopted by the Board of Direction, October 29, 1951.

Who's Who This Month

E. J. Critzas

E. J. Critzas, vice-president and construction engineer, Borsari Tank Corporation of America, New York, N. Y., and St. Louis, Mo., presents "Field Experiences in Building Multiple Story Reinforced Concrete Tanks," p. 365.

Mr. Critzas has been employed by the Borsari Tank Corporation of America since 1935, his duties including direct field supervision of construction of reinforced concrete tank buildings. He also conducted extensive research in prestressed concrete for two years under the direction of the late Herman Schorer.

Prior to joining the staff of the Borsari Tank Corporation, he designed and supervised construction of reinforced concrete structures in Athens, Greece, and in the New York City area.

Receiving a BS degree from Columbia University in 1917 and civil engineering degree in 1919, Mr. Critzas became assistant instructor at Columbia University. From 1942-1945 he lectured on graphic statics and strength of materials at University College, Washington University, St. Louis, Mo.

Mr. Critzas is an associate member of ASCE and has been an ACI member since 1942.

O. W. Irwin

O. W. Irwin, president, Rail Steel Bar Association, Chicago, Ill., discusses "Conservation of Steel by Design," p. 373.

A civil engineering graduate of the Case School of Applied Science, from 1906-1911 Mr. Irwin worked in the bridge and building department of the L.S. & M.S. Ry., Cleveland, Ohio. During this time he was responsible for the design of all bridges of the Ashtabula-Youngstown low grade and supervised the planning of the ore handling plant at Ashtabula, several freight houses and many main line bridge renewals and grade crossing eliminations. From 1911-1917, he was chief draftsman, Truscon Steel Co., Detroit, Mich., and Youngstown, Ohio, supervising the designing, estimating and detailing of hundreds of reinforced concrete structures.

Mr. Irwin is a life member of ASCE and has been affiliated with ACI since 1916.

David F. Stout and Robert E. Wilde

David F. Stout and Robert E. Wilde are the co-authors of "Automatic Jacks Speed Sliding-Form Construction," p. 381.

David F. Stout is superintendent of the Jack Construction Co., Sylvania, Ohio.

In 1940 he entered military service and served five years in the European and Pacific theaters. After separation from the U. S. Army, he worked for several contractors learning the business and nature of concrete construction and after 2½ years as field superintendent for the Missouri Farmers Assn., Columbia, Mo., Mr. Stout joined the Jack Construction Co. as a steel foreman, working in Illinois, Kansas, Colorado, Nebraska and Wisconsin. Presently, he is superintendent of a project near Ottawa Lake, Mich.

Robert E. Wilde is associate editor, *Rock Products*, Chicago, Ill. A civil engineering graduate of Iowa State College, March 1949, Mr. Wilde was associate editor of JOURNAL OF THE AMERICAN CONCRETE INSTITUTE from April 1949 to August 1951.

Mr. Wilde served with the U. S. Army Air Forces for over three years and presently is an officer in the Corps of Engineers, U. S. Army Reserve.

A member of ACI, he is also affiliated with ASCE and Sigma Delta Chi (professional journalism fraternity).

Harry W. Easterly, Jr.

"Shrinkage and Curing in Concrete Masonry Units," p. 393, introduces Harry W. Easterly, Jr., executive vice-president, Concrete Pipe and Products Co., Inc., Richmond, Va.

Receiving a BS in civil engineering from the Virginia Military Institute, Mr. Easterly joined the Concrete Pipe and Products Co., Inc., in 1947.

During World War II, he served for three years as an officer in the U. S. Marine Corps.

A member of ASTM Committees C-13 and C-15 and ACI Committee 716, Mr. Easterly is a junior member of ASCE and has been affiliated with ACI since 1948.

Robert B. B. Moorman

"Equivalent Load Method for Analyzing Prestressed Concrete Structures," p. 405, is contributed by Robert B. B. Moorman, professor of civil engineering, University of Missouri, Columbia, Mo.

Receiving a BS degree from the University of Illinois in 1929 and a MS degree in civil engineering in 1931, Dr. Moorman obtained his PhD in structural engineering from Iowa State College in 1937.

In 1931 he began his career at the University of Illinois as a special research assistant in theoretical and applied mechanics. From 1932-1935 he was professor of civil engineering at the University of Mississippi. In 1935 he joined the faculty of the University of Missouri where he has been since with the exception of three years during World War II when he was structures consultant with Goodyear Aircraft Corp.; later he was bridge engineer on an expressway with Consoer, Townsend and Associates, and then engineer on guided missiles at Johns Hopkins University Applied Physics Laboratory.

Concurrent with his teaching he has been associated with local architects on various projects, one of which was the structural design of a monolithic concrete armory, and at various times he has been associated with Sverdrup & Parcel, consulting engineers, St. Louis, Mo., on structural design and applied mechanics. During the summer months he worked on preliminary design with Boeing Aircraft Co., Seattle, Wash., and on structural design of concrete buildings and steel floating dry docks for the Navy Department Bureau of Yards and Docks.

A member of ACI since 1937, Dr. Moorman is also affiliated with ASCE, ASEE, International Association for Bridge and Structural Engineering, Sigma Xi, Tau Beta Pi, Sigma Tau and Chi Epsilon.

A. W. Brust

A. W. Brust, associate professor of civil engineering, Washington University, St. Louis, Mo., is the author of "Why Small Jobs Frequently Get Poor Concrete," p. 417.

A civil engineering graduate of the University of Nebraska, Professor Brust joined the staff of Washington University in 1924 and has been in charge of plain concrete and materials of engineering since 1933.

He is a consultant to several St. Louis ready-mix plants and in 1941 was concrete engineer of the St. Louis Ordnance Plant. During the summer of 1950, he was field engineer for PCA.

Professor Brust has been an ACI member since 1950.

Membership certificates

Repeated but infrequent requests for membership certificates indicate that enough members may want them to warrant having them made. At its fall meeting, the Board of Direction authorized the Secretary to determine through this notice whether or not the Institute would be justified in going to the expense of having the necessary work done.

It is estimated that certificates suitable for framing can be furnished ACI Members at a cost of not over $3.50. Those interested are requested to clip the attached coupon and mail it to Institute headquarters. *Please do not send money until billed.* If there are not enough interested, the Institute would not be justified in supplying certificates to a few at a loss.

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Detroit 19, Mich.

Gentlemen: Please place an order for one membership certificate to be furnished provided enough members buy them to make it worthwhile at a cost not to exceed $3.50.

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West Coast Conferences on Practical Concrete Problems

By Lee J. Rothgery*

Four very successful conferences on practical concrete problems have been held in California during the past year.

The first of these conferences was held in San Francisco, May 4, 1951, under the joint auspices of the University of California Engineering Extension Division, PCA Information Bureau, AGC and the west coast membership of ACI. Nearly 300 attended, most of them bristling with questions. Because questions and answers were not to be published, problems of a controversial nature were discussed. To lead the discussion, E. L. Howard, J. E. Jellick and J. W. Kelly were among the capable panel of men representing a cross section of the concrete industry of the San Francisco area.

The reactions of those who attended the conference were unquestionably favorable. They were very much interested and actively so. There was evidence that the conference filled a definite need and there were indications of a desire for more sessions of this kind.

Apparently interest in this type of conference was not limited to the San Francisco area. On June 22, the Fresno area arranged a conference which had an attendance of 200. Professor Kelly and Messrs. Jellick and Howard, with the experience gained at San Francisco, served again on the panel at Fresno. The engineers, architects and contractors associations, Cement Finishers' Union, Builders' Exchange, transit mix groups and the PCA Information Bureau cooperated with the University of California Engineering Extension Division in presenting this conference.

With the success of these conferences in northern California, the University of California Engineering Extension Division sounded out the sentiment for similar sessions in the southern part of the state. The response was most encouraging and conferences were arranged at San Diego on October 25 and at Los Angeles on October 26. In arranging these meetings, the Extension Division had the cooperation of PCA, AGA, the concrete industries and the ocal membership of ACI.

Interest in these southern conferences was evidenced by the attendance. In San Diego 140 were present in spite of the competition of another important engineering meeting while 400 attended at Los Angeles.

The idea of the "practical problems" type of conference is highly recommended for other sections of the country. Other regional groups of ACI who are looking for educational activities would find such sessions popular.

*Associate Engineer, Institute of Transportation and Traffic Engineering, Univ. of Calif., Berkeley, Calif.

Positions and Projects — ACI Members

Magnel pays tribute to Freyssinet

In the inaugural address of the International Congress of Prestressed Concrete, Ghent, Belgium, Prof. Gustave Magnel paid tribute to Eugene Freyssinet.

He cited the history of the advancements made in reinforced concrete and the important influence that Mr. Freyssinet had on modern construction practice.

Concluding his talk, Professor Magnel presented Mr. Freyssinet with a certificate and medal as tokens of honorary membership in A.I.G.

Mumford appointed vice-president

Russell P. Mumford has been appointed vice-president and general manager of The Beckley and Meyers Co., concrete products manufacturers, Springfield, Ohio.

Instituto Tecnico de la Construccion y del Cemento

The Instituto Tecnico de la Construccion y del Cemento, Madrid, Spain, under the direction of E. Torroja, covers research into all aspects of building, civil engineering, and conducts specialized research into the manufacture and uses of cement.

The Institute is making a number of practical tests on new methods of construction and new materials and devotes much time to fundamental research on the theory of elasticity, new mathematical techniques of calculation, etc.

An important side of the work of this organization is the collection of up-to-date information on all matters related to construction, and of making this data available. In addition to reports of latest developments, the Institute issues a monthly magazine providing general and specialized information on building and architecture, and a series of monographs dealing with particular aspects of research.

To stimulate original thought and discover the best possible solution to construction problems, the Institute organizes a number of national and international competitions, one of which is an award for the design of inexpensive dwellings.

Clair honored

At the recent 60th anniversary of the Drexel Institute of Technology, Miles N. Clair, 1921 graduate of DIT, was awarded a citation for civic leadership, for studies at the Logan International Airport, Boston, and for advancing the technology of concrete construction.

Mr. Clair is president of Thompson and Lichtner Co., Inc., Brookline, Mass., and has been affiliated with ACI since 1925.

ASA announces meeting plans

The Third National Standardization Conference, sponsored by the American Standards Assn., will be held in conjunction with the Centennial of Engineering in Chicago, September 8-10, 1952. The three-day series of meetings, emphasizing the role of standards in the many technological advances during the past century, will take place at the Museum of Science and Industry.

ITTE plans highway conference in Los Angeles

The Institute of Transportation and Traffic Engineering and the University Extension, University of California, are planning the Fourth Annual California Street and Highway Conference to be held at the UCLA campus, February 6-8, 1952.

Accommodations are being arranged at the Hollywood Roosevelt Hotel with special busses to and from the University campus.

ASTM committees meet

Through arrangements made by Prof. K. B. Woods, chairman of ASTM Committee C-9, the Purdue University campus was the scene of the combined 1951 fall meetings of ASTM Committees C-1 on Cement and C-9 on Concrete and Concrete Aggregates.

Both committees are planning to meet during ASTM Committee Week in Cleveland, Ohio, March 3-7, 1952, and at the 50th Anniversary Annual Meeting in New York, N. Y., June 23-27, 1952, at which time a symposium on factors affecting durability of concrete will be sponsored by Committee C-9.

Georgia Institute of Technology short course

The Georgia Institute of Technology's second annual quality concrete short course will be held February 4-6, 1952, in Atlanta, Ga. Intended to bring members of the construction industry up to date on the design and control of concrete mixtures, placing, forms, and curing, the course offers actual laboratory participation and comprehensive instruction from representatives of qualified organizations.

Cooperating associations for the short course are: ACPA, Atlanta section of AIA, Georgia section of ASCE, Georgia section of AGC, Georgia Engineering Society, Georgia Society of Professional Engineers, NRMCA, PCA, Atlanta Post of the Society of American Military Engineers, Georgia Concrete Pipe Mfg. Assn., and NCMA.

Inquiries regarding registration should be addressed to R. E. Eskew, acting coordinator, Short Courses and Conferences, Engineering Extension Div., Georgia Institute of Technology, Atlanta, Ga.

Edward J. Nunan

Edward J. Nunan has been appointed materials engineer and sales manager for the New York area of Buffalo Slag Co., Inc., Federal Crushed Stone Corp., and Hornell Gravel Corp., Buffalo, N. Y.

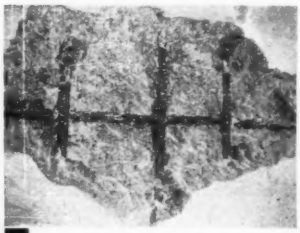
Zollman accepts new position

Charles C. Zollman recently became associated with Vacuum Concrete, Inc., Philadelphia, Pa., as chief engineer. Formerly he held the same position with Prestressed Concrete Corp., Kansas City, Mo.

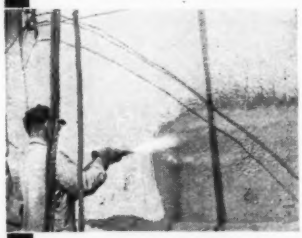
Victor A. Malmberg

Victor A. Malmberg, ACI member since 1942, died recently at Veterans Hospital, St. Cloud, Minn.

Mr. Malmberg was a civil engineering graduate of the University of Minnesota in 1920. A registered civil engineer in Minnesota, he was employed by the U. S. Engineers in St. Paul, Minn., for a number of years.



Typical example of spalling. Note corrosion of reinforcing rods exposed by disintegrating concrete.



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New Members

The Board of Direction approved 60 new member applicants for November—36 Individual, 1 Corporation, 17 Junior and 6 Student. With adjustments made for a few losses from resignations, deaths and nonpayment of dues, the membership total became 5448 on December 1, 1951.

Individual

BIEBER, HARRY, Los Angeles, Calif. (Struct. Design, Proj. Engr., The Austin Co.)
BUTLER, GORDON H., Duluth, Minn. (Owner, Pres., Polaris Concrete Products Co.)
CASHDAN, DANIEL MICHAEL, Bronx, N. Y. (Struct. Design, Brown-Guenther-Booss)
CLAUS, ROGER JOHN, Lansing, Mich. (Instructor, Mich. State College)
COVALT, W. L., Kokomo, Ind. (Pres., Genl. Mgr., Kokomo Ready Mixed Concrete Corp.)
DELBOUIS, ALFREDO LEON, Mexico D. F., Mexico (Concrete Supt., Ingenieros Civiles Asociados S. A. de C. V. Contracting & Constructing Co.)
DESIMONE, SALVATORE V., New York, N. Y. (Struct. Design, Brown-Guenther-Booss)
DUCHARME, JEAN MARC, Needham, Mass. (Pres., Chf. Engr., Crandall Dry Dock Engrs., Inc.)
EMMONS, WALTER F., Norris, Tenn. (Super., TVA)
ENGLE, WARD W., Knoxville, Tenn. (Civil Design Engr., TVA)
FRUCHER, PIERRE, New York, N. Y. (Design, Brown-Guenther-Booss)
GAUS, FRANK D., Long Beach, Calif. (Salesman, Dewey & Almy Chem. Co.)
GRANDOS S., ENRIQUE DIAZ, Barranquilla, Colombia, (Struct. Engr.)
GRANTHEM, KENNETH N., San Francisco, Calif. (Grad. St., Univ. of Calif.)
HAMRICK, SAMUEL J., Terre Haute, Ind. (Genl. Craft Supt., The Girdler Corp.)
HEDMAN, AAKE F., Chattanooga, Tenn. (Cons. Engr.)
HOWE, LYALL F., Toorak, Vic., Australia (Asst. Project Officer, Dept. of Works & Housing)
JAMIAN, S. C., Riverdale, Md. (Head of Design Sect., Washington Dist. Office, Corps of Engrs.)
JEWELL, K. AUSTIN, Evanston, Ill. (Hydraulic Design Engr., Koppers Co.)
JONES, RAYMOND M., Washington, D. C. (Instr., Howard Univ.)
KLEINMAN, ADAM, Tel-Aviv, Israel (Struct. Design, Inspection, M. Tintner, Arch.)
LACY, FLOYD B., JR., Knoxville, Tenn. (Super, TVA)
MEIER, LOUIS J., Knoxville, Tenn. (Design, TVA)
MOLINA R., CARLOS, Roma, Texas (Resident Engr., Mexican Govt.)
MOORE, WALTER P., Houston, Texas (Cons. Engr.)
NICHOLS, GEORGE T., Jackson, Miss. (Design and Control Super., Jackson Ready-Mix Concrete)
SACHER, G. ERIC, Roanoke, Va. (Resident Engr., Assoc. Housing Consultants; Inspection Engr.)
SCHLEGEL, HENRY K., San Francisco, Calif. (Chf. Struct. Engr., Bechtel Corp.)
SEALE, J. J., Chico, Calif. (Valley Concrete Pipe & Products Co., Inc.)
SHEDD, JACK P., Manhattan, Kans. (Asst. Prof., Kansas State College)
SHHABEL, M., Tel-Aviv, Israel (Struct. Engr., A. Leitersdorf & I. Belzitzman, Archs.)
SMITH, JOHN W., Powell, Tenn. (Design, TVA)
SPURGEON, W. M., Cincinnati, Ohio (Research Dir., American Fluresit Co.)
TOLNAFY, S. B., Footscray, Australia (Arch., Commonwealth of Australia)
WITTES, DAVID ROBERT, New York, N. Y. (Super., Struct. Design, Brown-Guenther-Booss)
WOODYARD, J. F., JR., Monterrey, Mexico (Arch., Engr., Contr., Construcciones Woodyard)

Corporation

SOUTHERN LIGHTWEIGHT AGGREGATE CORP., Richmond, Va. (A. C. Ford)

Junior

BAKER, WALTER F., Brentwood, Mo. (Design, Drafting, Concrete Products Mfg. Co.)
BECK, CYRIL MELVIN, Halifax, N. S., Canada (Arch. Representative, Fetherstonhaugh, Durnford, Bolton & Chadwick)
BIRNBAUM, HAROLD, Irvington, N. J. (Struct. Engr., Ammann & Whitney)
FREEMAN, SIMOND DAVID, Knoxville, Tenn. (Design, TVA)
GAMBONE, JOSEPH L., Bronx, N. Y. (Struct. Design, Brown-Guenther-Booss)
HAKEDA, TSUTOMU, Wahiawa, Oahu, Hawaii (Civil & Struct. Engr., Area Engrs.)
HILDEBRAND, JOHN R., Salem, Va. (Design, Drafting, Mattern & Mattern, Engrs.)
HOFFMAN, ALBERT A., Jr., Berkeley, Calif. (Instructor, Univ. of Calif.)
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STRUBINGER, TAYLOR E., University City, Mo. (Field Engr., Merritt-Chapman & Scott)
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Student

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Honor Roll

February 1—November 30, 1951

The Honor Roll for the current year ends January 31. A complete list including the 50-50 credits will be published in the March Journal. Make it a point to have your name on this list of members having furthered the Institute's efforts to bring the knowledge of concrete to all who are interested!

Newlin D. Morgan, Sr. (Ill.)	40
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Frank Kerekes (Ia.)	4
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I. E. Morris (Ga.)	3
C. P. Siess (Ill.)	3
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E. L. Howard (Calif.)	2½
T. C. Kavanagh (Pa.)	2½
F. T. Mavis (Pa.)	2½
Donald M. Agrimson (S. D.)	2
Boyd G. Anderson (N. Y.)	2
J. F. Baxter (N. Y.)	2
E. J. Critzas (Mo.)	2
Robert G. Deitrich (Md.)	2
Frank L. Ehasz (N. Y.)	2
H. F. Gonnerman (Ill.)	2
H. R. Harbeson (Mich.)	2
Henry L. Kennedy (Mass.)	2
David R. McKenzie (Brazil)	2
Thomas J. Reading (S. D.)	2
Dana A. Scheid (Ind.)	2
Howard Simpson (Mass.)	2
Harry F. Thomson (Ill.)	2
LeRoy A. Thorssen (Canada)	2

ECA sponsors dam project in Malaya

American geological and engineering know-how is being exported by the Economic Co-operation Administration to help provide a much needed new water supply for the capital of the Federation of Malaya.

Two U. S. Bureau of Reclamation experts, Edward R. Dexter, civil engineer, and Dr. William H. Irwin, geologist, left the United States recently for the Federation Capital, Kuala Lumpur, where they will study the proposed construction of a reservoir dam nearby.

The present water supply, with a capacity of 10.25 million gallons per day, is inadequate for the needs of the estimated 230,000 persons in and around the capital city served by this source. Surveys during the past few years have indicated that the best possibility for increasing the water sources would be by constructing an impounding dam in the Klang River at the Klang Gates, a few miles east of Kuala Lumpur.

The latest investigations which included the sinking of a shaft in the river bed and driving a tunnel under it, show this is the best site for a high dam. However, the quartz foundation is fissured to some extent, presenting problems of construction and effective operation of the proposed dam.

Authorities believe the dam can be built, but a number of questions regarding the foundation, prevention of leakage, materials problems and construction procedure still exist.

The request for American technical assistance was made because available British engineers have had little experience in the construction of high dams in similar geological formations. Such conditions exist at some dam sites in the United States, including the Kentucky Dam of the Tennessee Valley Authority system.

Mr. Dexter and Dr. Irwin will be in Kuala Lumpur for about six weeks on the study and will make their recommendations to the Federation government. They were selected by the Bureau of Reclamation and ECA because of their experience in such site conditions. Dr. Irwin has been an ACI member since 1947.

Brundage honored

Avery Brundage was among the 100 persons honored with "Centennial Awards for the Northwest Territory" by Northwestern University. The awards were given to leaders in arts, science, industry and government from Illinois, Indiana, Michigan, Ohio, Wisconsin and Minnesota in recognition of "the impress they had made upon their generation during a lifetime of distinguished service as residents of one of the states which comprised the original Northwest Territory."

Mr. Brundage, president, Avery Brundage Co., general contractors, Chicago, Ill., has been affiliated with ACI since 1925.

Short course on aggregates and concrete

The Sixth Annual Short Course of Instruction on Aggregates and Concrete sponsored by the University of Maryland, National Sand and Gravel Assn., and National Ready Mixed Concrete Assn., was held November 12-16, 1951, at the University of Maryland, College Park, Md.

Ninety-eight attended this short course where topics discussed included problems of field control of concrete, factors affecting strength and consistency of concrete, air-entraining concrete, design of concrete mixtures, aggregate specifications and plant operating problems.

ALPHABETICAL LIST OF ADVERTISERS

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Tools, Materials, Services

Under this heading note will be made from time to time of producer literature of presumed technical interest (and available from its source for the asking) to ACI users of tools, equipment, materials, accessories and special services.

Smooth ceilings system

A new illustrated bulletin describing theory, application and benefits of the smooth ceilings system of slab construction is now available. Installations described include office buildings, industrial buildings, hospitals, schools, garages and dormitories. Tables based on standard building code formulas are provided to assist building designers in selecting slab thickness.

Copies of the new bulletin may be obtained upon request.—Smooth Ceilings System, 802 Metropolitan Life Bldg., Minneapolis 1, Minn.

Modern placement of concrete

A new and enlargeed dition of the Protex Modern Placement of Concrete fact book, has been published recently by Autolene Lubricants Co., Denver, Colo.

Published under the direction of Autolene Industrial Research Division, the new edition contains the latest technical information and field use tips on air-entraining concrete. Complete with photographs of the latest concrete construction projects, the book is full of questions and answers about air-entrainment technique and Protex air-entraining solutions.

Many new photographs have been included to show actual test pictures of concrete with and without Protex air-entrainment solutions. The photographs graphically show the advantage of air-entrainment in structural work. Other photos show construction projects. Many new graphs and charts have been added in the new edition.

Copies of the new edition of the MPC booklet may be had upon request.—Autolene Lubricants Co. Industrial & Research Div., Denver 9, Colo.

DISCUSSION

Discussion closed January 1, 1952

Sept. Jl. '51

Earthquake Resistant Design Considerations—R. R. Martel

Comprehensive Numerical Method for the Analysis of Earthquake Resistant Structures—Charles S. Whitney, Boyd G. Anderson and Mario G. Salvadori

Multistory Buildings Designed to Resist Earthquakes—John J. Gould

Lightweight Concrete for Lower Construction Costs—J. A. Murlin

Designing for Continuity in Prestressed Concrete Structures—Alfred L. Parme and George H. Paris

Pumice—Lightweight Aggregate—Leslie I. Neher

Hydration Products Formed in Cement Pastes at 25 to 175 C—George L. Kalousek and Milton Adams

Title No. 48-31

Use of Chicago Fly Ash in Reducing Cement-Aggregate Reaction*

By C. H. SCHOLER and G. M. SMITH†

SYNOPSIS

The use of 20 to 30 percent Chicago fly ash, by total weight of fly ash plus cement, effectively and economically inhibits certain types of cement-aggregate reaction in concrete. Fly ash, a finely divided dry powder collected by precipitators from flue gases of pulverized coal-burning power plants, forms a cementing medium when it combines with the lime liberated during the hydration of a portland cement. The data presented in this paper refer only to a fly ash obtained from the Chicago area; fly ash from other localities may not necessarily produce the same results since they are known to vary considerably in chemical and physical properties.

Laboratory tests consisted of subjecting 3 x 4 x 16-in. beams to two different accelerated exposures to determine cement-aggregate compatibility. These two accelerated exposures have shown an excellent correlation with observed field service records. Cement-aggregate reaction as referred to in this paper is not restricted to the alkali-aggregate reaction, but refers to the physical and perhaps chemical reactions or a combination of both which causes the expansion that is accompanied by "map cracking."

Use of fly ash produces concretes similar in nature to concretes with ordinary portland cement, with fly ash concretes showing an increase in workability and improved finishing characteristics.

INTRODUCTION

Cement-aggregate reaction has been a menacing cause of deterioration of concrete throughout Kansas, Missouri, Iowa, Nebraska, and other midwest localities where sand-gravel is used extensively because of the scarcity of coarse aggregates. Sand-gravel aggregate is the sand and gravel containing little coarse material obtained from the local rivers and their tributaries.

An extensive series of laboratory tests, conducted to find a satisfactory means for inhibiting cement-aggregate reaction, indicates that fly ash does so economically and effectively. C. H. Scholer and W. E. Gibson[1] found that certain coarse aggregates were also effective in reducing the amount of cement-aggregate reaction but where coarse aggregates are not locally available, their use is not economical.

*Received by the Institute Sept. 27, 1951. Scheduled to be presented at the ACI 48th Annual Convention, Cincinnati, Ohio, Feb. 26-28, 1952. Title No. 48-31 is a part of the copyrighted JOURNAL OF THE AMERICAN CONCRETE INSTITUTE, V. 23, No. 6, Feb. 1952, *Proceedings* V. 48. Separate prints are available at 35 cents each. **Discussion** (copies in **triplicate**) should reach the Institute not later than June 1, 1952. Address 18263 W. McNichols Rd., Detroit 19, Mich.

†Members American Concrete Institute, Professor and Assistant Professor, Applied Mechanics, respectively, Kansas State College, Manhattan, Kans.

METHOD OF TEST

The two accelerated tests used were a wetting and drying test, and a continuous spray of water, with a temperature change occurring in each. The wetting and drying test, referred to here as Exposure 2, consists of drying saturated specimens of concrete in ovens at 130 F for 8 hours, followed by immersion in water at an initial temperature of 70 to 80 F for 16 hours. The process is repeated 5 days each week, the specimens remaining immersed over Saturday and Sunday.

The continuous spray of water, referred to as Exposure 10, consists of placing the beams, standing on end, in a tank where they are exposed to a continual spray of water at a rather low velocity but of such intensity and distribution that water is continually falling on all surfaces of the beams. One complete cycle consists of changing the temperature of the water from 68 to 135 F, and then back down to 68 F. This cycle gives a temperature range at the center of the beam of 74 to 122 F. The temperature change is obtained automatically, and is gradual so as to eliminate any "shock" effect. The automatic controls are so set as to obtain 32 cycles per day or 224 cycles per week. The spray tank installation is shown in Fig. 1. Exposure 10 was developed under a cooperative research program between the Portland Cement Assn. and the Engineering Experiment Station at Kansas State College.

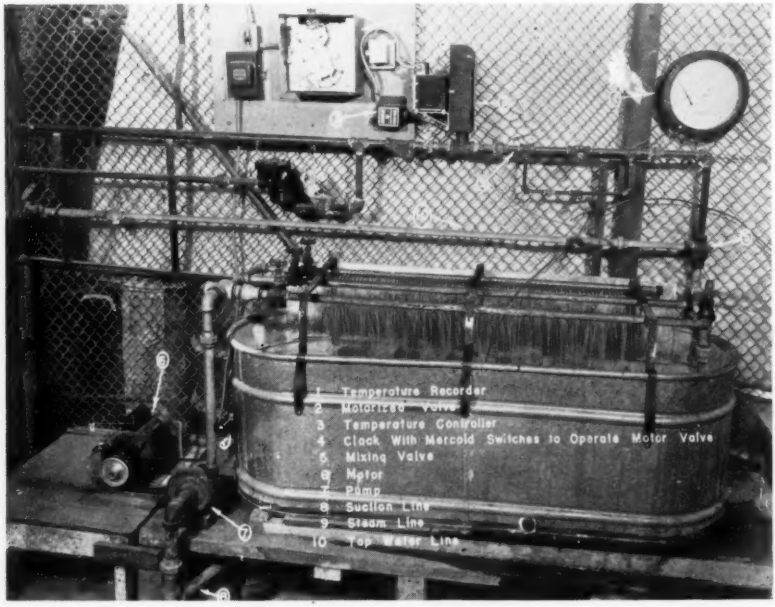

Fig. 1—Spray tank installation (exposure 10)

The specimens were 3 x 4 x 16-in. beams provided with gage points in each end. All concrete beams were cured 24 hours in the molds in the moist closet, and six days in the moist room. At the age of 7 days, the beams were removed from the moist room, and stored on racks in the air-conditioned laboratory at 74 F, and at 55 to 60 percent relative humidity for 21 days. All specimens were then placed in water at 70 F for two days before the exposure tests were started at 30 days age. For mixing data see Table 1.

In addition to the normal curing, one cement and aggregate combination with 0, 20, and 30 percent substitutions* of fly ash were cured at a low temperature to determine whether the fly ash would still be effective in reducing cement-aggregate reaction in Exposure 2. These specimens were cured one day in the molds in the moist room, followed by 27 days at 42 F dry

*Substitutions are based upon total weight of the cementing medium with fly ash being considered as part of the cement.

TABLE 1—MIXING DATA

Material	Fly ash, percent	W/C, gal. per sack	Cement factor,* bbl per cu yd	Air, percent	Slump, in.
Cement A					
Eau Claire sand	0	5.95	1.88	5.0	1.25
Eau Claire sand	20	6.40	1.89	4.0	2.50
Eau Claire sand	30	6.15	1.89	3.6	2.25
Platte River sand	0	5.44	1.59	3.0	2.00
Platte River sand	20	5.55	1.60	2.0	3.00
Platte River sand	30	5.27	1.61	1.9	3.00
50 percent Platte River sand + 50 percent quartzite	0	5.65	1.64	1.8	2.50
	20	5.26	1.64	1.0	2.50
	30	5.20	1.64	1.0	3.00
50 percent Platte River sand + 50 percent dolomite	0	5.50	1.63	2.1	2.00
	20	4.93	1.64	1.0	2.75
	30	4.78	1.64	2.2	2.25
50 percent Platte River sand + 50 percent Eau Claire gravel	0	4.65	1.67	1.6	4.00
	20	4.50	1.68	1.0	2.50
	30	4.82	1.66	0.5	4.00
Cement B					
Platte River sand	0	5.90	1.55	4.0	1.75
Platte River sand	20	5.37	1.60	2.0	3.00
Platte River sand	30	5.15	1.55	2.2	2.75
50 percent Platte River sand + 50 percent quartzite	0	5.66	1.61	2.2	3.00
	20	5.45	1.65	1.1	2.00
	30	4.94	1.66	1.3	2.50
50 percent Platte River sand + 50 percent dolomite	0	5.20	1.63	2.5	1.75
	20	5.02	1.65	1.3	3.00
	30	4.87	1.63	2.0	2.75
50 percent Platte River sand + 50 percent Eau Claire gravel	0	4.75	1.67	2.0	2.25
	20	4.85	1.67	1.3	2.50
	30	4.72	1.68	1.3	3.00
Cement C					
Eau Claire sand	0	6.68	1.86	5.0	2.75
Eau Claire sand	20	6.20	1.89	4.2	2.40
Eau Claire sand	30	6.20	1.85	3.2	3.00

*Fly ash substitution is considered to be a part of the cement.

TABLE 2—CHEMICAL ANALYSIS OF CEMENTS

Cement	SiO_2	Al_2O_3	Fe_2O_3	CaO	MgO	SO_3	Loss on ign.	K_2O	Na_2O	NA_2O equiv., total alkali
A	20.90	6.55	2.55	63.63	1.96	1.89	1.55	0.21	0.28	0.42
B	21.00	5.78	2.49	63.30	3.52	2.03	1.22	0.74	0.16	0.65
C	20.77	5.90	2.82	63.23	2.77	2.30	1.07	0.59	0.28	0.67

storage. At 28 days' age the specimens were then placed in water at 70 F for two days before starting the Exposure 2 tests at the age of 30 days.

A comparison of wetting and drying tests with a field survey made by White,[2] showed that cement-aggregate combinations that expand as much as 0.07 percent or more in one year in the wetting and drying tests have a poor service record.

MATERIALS

Three brands of portland cement (Type I) were used, two of which (B and C) have been used extensively in the Kansas and Nebraska area, and the other (A) in the Chicago area. See Table 2 for chemical analyses.

The sands used were a Platte River sand, a Republican River sand and a sand from Eau Claire, Wis. The Platte and Republican River sands are known to be reactive, and have a poor service record, while Eau Claire sand is considered to be nonreactive, and has a good service record.

To study the effect of fly ash on concretes containing coarse aggregates, a Chicago dolomite, an Eau Claire gravel, and a Kansas quartzite were used in combination with the Platte River sand.

The fly ash used in this program was obtained from a power plant in the Chicago area through the Combustion By-Products Co. See Table 3 for chemical and physical test results.

There are many clays and shales that when calcined and finely ground can be used as pozzolanic material.* To compare the fly ash with some other pozzolans, a calcined Monterey shale from California and a calcined Mowry shale from Wyoming were used with one of the cements and a Republican River sand.

TABLE 3—CHEMICAL AND PHYSICAL CHARACTERISTICS OF CHICAGO FLY ASH

Chemical composition, percent			
Silica	46.4	Titanium	0.4
Alumina	14.8	Total Alkalies	3.3
Iron oxides	24.9	Sulfuric anhydride	2.4
Lime	4.1	Loss on ignition	2.6
Magnesia	0.5	Carbon	1.5

Physical data	
Sieve analysis	96.4 passing a 325-mesh sieve
Blaine surface area	3380 sq cm per g
Specific gravity	2.51

RESULTS OF EXPOSURE 2

Fig. 2 shows the percent expansion occurring in the wetting and drying tests (Exposure 2) for combinations of cements and sand-gravels. The ex-

*A pozzolan is a siliceous material which possesses no cementitious value in itself, but, when finely divided and in the presence of moisture, will react chemically with calcium hydroxide to form a cementitious compound.

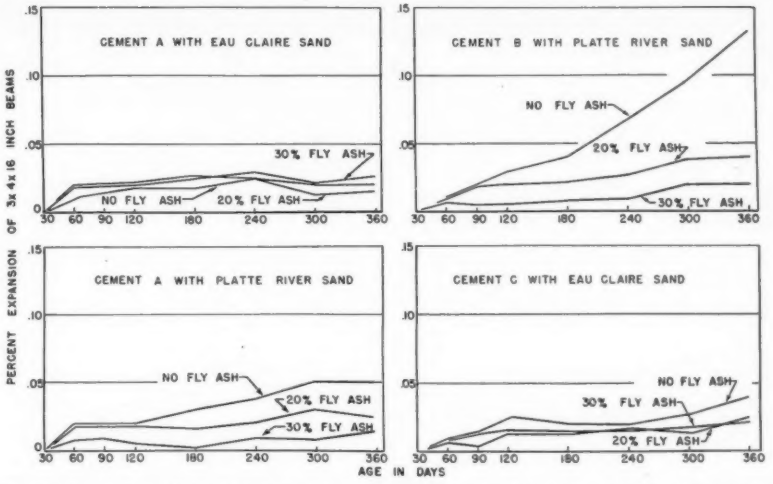
CEMENT A WITH EAU CLAIRE SAND
CEMENT B WITH PLATTE RIVER SAND
CEMENT A WITH PLATTE RIVER SAND
CEMENT C WITH EAU CLAIRE SAND
NO FLY ASH
20% FLY ASH
30% FLY ASH
PERCENT EXPANSION OF 3x4x16 INCH BEAMS
AGE IN DAYS

Fig. 2—Expansion obtained from alternate cycles of heating and drying and cooling and soaking (exposure 2)

pansion of cement B and Platte River sand indicates a high degree of cement-aggregate reaction, but the substitution of 20 percent fly ash virtually eliminated the expansion so that little, if any, deterioration would be expected. Substitution of 30 percent fly ash is even more effective in reducing expansion. The expansion (0.05 percent) of Cement A with Platte River sand indicates only slight reactivity, and deterioration in the field would probably be very slow. The use of fly ash should be given serious consideration as a possible means for obtaining long life from concretes which would otherwise deteriorate as a result of excessive or moderate cement-aggregate reaction.

Apparently Eau Claire sand when combined with either Cements A or C, is nonreactive, and the substitution of fly ash with these combinations has no effect in increasing or decreasing the expansion.

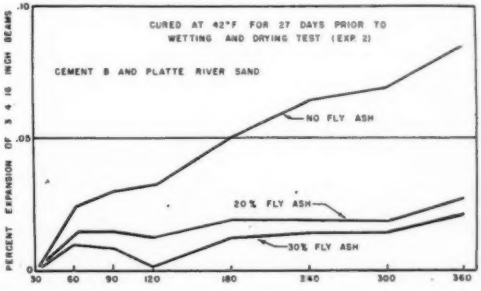
CURED AT 42°F FOR 27 DAYS PRIOR TO WETTING AND DRYING TEST (EXP. 2)
CEMENT B AND PLATTE RIVER SAND
NO FLY ASH
20% FLY ASH
30% FLY ASH
PERCENT EXPANSION OF 3 4 16 INCH BEAMS

Fig. 3—Effect of Chicago fly ash on the expansion of concrete cured at low temperatures

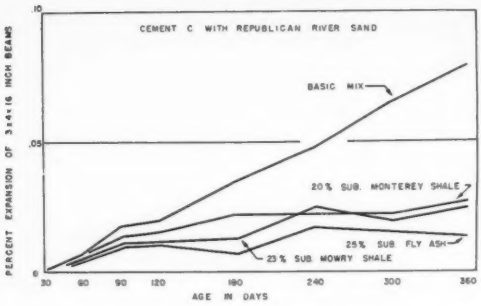
CEMENT C WITH REPUBLICAN RIVER SAND
BASIC MIX
20% SUB. MONTEREY SHALE
25% SUB. FLY ASH
23% SUB. MOWRY SHALE
PERCENT EXPANSION OF 3x4x16 INCH BEAMS
AGE IN DAYS

Fig. 4—Effect of Chicago fly ash, Monterey shale and Mowry shale on expansion obtained from exposure 2

The authors have been asked several times if fly ash effectively reduces cement-aggregate reaction in concrete cured at low temperatures. Fig. 3 shows the effect of fly ash on the expansion of Cement B and Platte River sand, when initially cured at 42 F for 27 days, followed by 48 hours of soaking, before being subjected to the wetting and drying test. It is apparent in Fig. 3 that the low temperature curing, prior to the wetting and drying tests, did not inhibit the pozzolanic action of the fly ash.

Fig. 4 shows the effect of Mowry shale, Monterey shale, and fly ash as being about equally effective in reducing the cement-aggregate reaction of Cement C and a Republican River sand in the wetting and drying test.

The concretes containing the coarse aggregates developed so little expansion in the wetting and drying tests (Exposure 2) and the continuous spray of water (Exposure 10), that the results have been omitted here. So

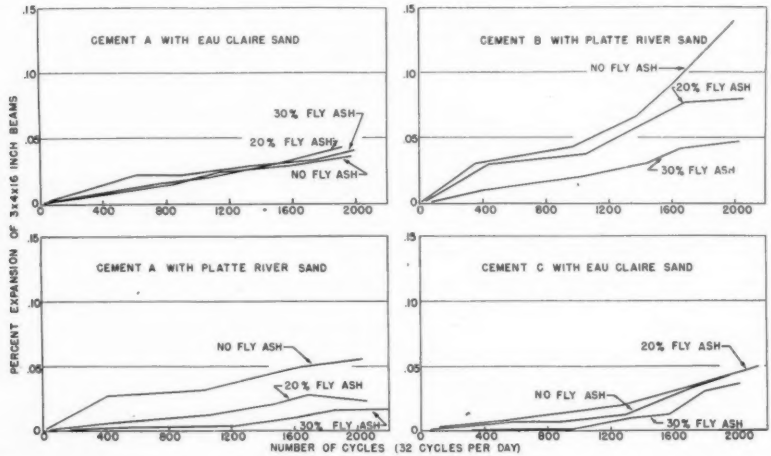
CEMENT A WITH EAU CLAIRE SAND
30% FLY ASH
20% FLY ASH
NO FLY ASH
CEMENT B WITH PLATTE RIVER SAND
NO FLY ASH
20% FLY ASH
30% FLY ASH
CEMENT A WITH PLATTE RIVER SAND
NO FLY ASH
20% FLY ASH
30% FLY ASH
CEMENT C WITH EAU CLAIRE SAND
20% FLY ASH
NO FLY ASH
30% FLY ASH
PERCENT EXPANSION OF 3x4x16 INCH BEAMS
NUMBER OF CYCLES (32 CYCLES PER DAY)

Fig. 5—Expansion obtained from heating and cooling by a continuous spray of water (exposure 10)

little expansion occurred that no differences were indicated between these various mixes by the inclusion of fly ash.

RESULTS OF EXPOSURE 10

Fig. 5 shows that the relative expansions for the various concretes, submitted to the continuous spray of water (Exposure 10), are almost identical to the expansions occurring in the wetting and drying test (Exposure 2). It is difficult to determine the merit of an accelerated test that is to simulate field conditions, but it does seem highly significant that identical results were obtained in Exposures 10 and 2.

MODULUS OF RUPTURE AND COMPRESSIVE STRENGTHS BEFORE AND AFTER EXPOSURE 2

Fig. 6 shows the effect of fly ash on the modulus of rupture and compressive strengths at the age of 30 days, and after the wetting and drying tests. The strength tests cover a variety of sand-gravel and coarse aggregate combinations with Cements A and B. In general, the modulus of rupture and compressive strengths of the concretes containing 20 and 30 percent fly ash are slightly lower at 30 days than the corresponding concretes containing no fly ash. However, after receiving the wetting and drying tests, the concretes containing fly ash show a rather large and significant increase in both modulus of rupture and compressive strength over corresponding concretes with no

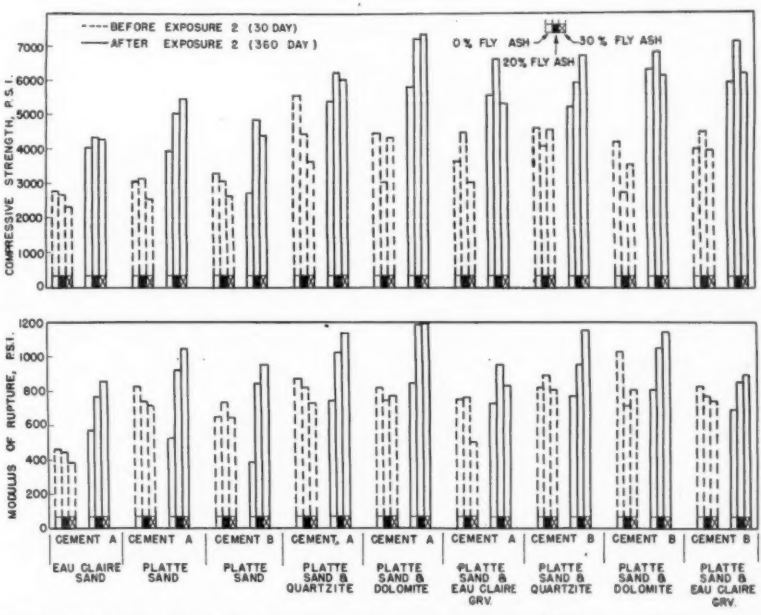

Fig. 6—Modulus of rupture and compressive strengths of concretes at 30 days and after exposure to wetting and drying (exposure 2)

fly ash. In several instances the 30 percent substitution of fly ash produced concretes having a modulus of rupture of well over 1100 psi.

DISCUSSION

It is not entirely understood just why fly ash reduces the expansion of concretes containing cements and aggregates that are not compatible. Perhaps a study of the physical and chemical properties of cement pastes containing fly ash would reveal some fundamental concepts as to the cause of cement-aggregate reaction. As an example, the thermal properties of cement pastes containing fly ash are considerably different from the thermal properties of cement pastes containing no fly ash. G. M. Smith[3] found that the thermal coefficient of expansion of ordinary portland cement paste ranged from 7.3×10^{-6} to 11.5×10^{-6} per deg F depending upon the degree of saturation, while pastes containing 30 percent Chicago fly ash, ranged from 5.2×10^{-6} to 9.0×10^{-6} per deg F. Undoubtedly plastic flow and many other physical properties of cement pastes are altered by addition of fly ash.

CONCLUSIONS

These conclusions are based upon the limited number of tests mentioned above and rather extensive tests made in connection with other projects involving the use of Chicago fly ash with several other cements and aggregates exhibiting definite cement-aggregate reaction.

In any proposal for the use of any fly ash with a cement-aggregate combination, it is urged that the user make suitable advance studies to determine the effectiveness of the proposed fly ash in preventing deleterious reaction.

(1) Twenty percent fly ash (by weight of cement plus fly ash) is sufficient to inhibit expansion for most reactive sands and cements, but 30 percent is recommended to reduce excessive reaction between cement and aggregate.

(2) In general, the use of 20 to 30 percent fly ash slightly reduces strengths at early ages, but when exposed to the accelerated wetting and drying tests, which simulate outside weathering, the strengths are far superior to corresponding concretes containing no fly ash.

(3) The use of 20 or 30 percent fly ash does not materially modify the usual characteristics of the freshly-mixed concrete, but improves the workability and finishing properties.

(4) The mix data in Table 1 indicate that the fly ash reduces the air content; hence, additional air-entraining agent may be required for proper air-entrainment in concrete containing fly ash.

REFERENCES

1. Scholer, C. H., and Gibson, W. E., "Effect of Various Coarse Aggregates Upon the Cement-Aggregate Reaction," ACI JOURNAL, June 1948, *Proc.* V. 44, 1009.

2. White, L. V., "Inspection of Concrete Structures, Republican and Blue River Watersheds," Unpublished report, Engineering Experiment Station, Kansas State College, 122 pp., 1941.

3. Smith, G. M., "The Effect of Size and Thermal Expansion of Aggregates on the Durability of Concrete," Unpublished Thesis, Applied Mechanics Dept., Kansas State College, 30 pp., 1951.

Title No. 48-32

Direct Design of Rectangular Columns with Bending About an Axis of Symmetry*

By HENRY J. COWAN†

SYNOPSIS

A new method for the design of eccentrically loaded reinforced concrete columns based on the principle of superposition. The section is designed to resist the bending moment due to the eccentricity of the column load, using the well-known equations for rectangular reinforced concrete beams; the area of reinforcement is then reduced to allow for the direct compression due to the column load.

In the standard method the dimensions of the section are assumed and the depth of the neutral axis and the maximum stresses in the steel and the concrete are then calculated. In the present method the neutral axis depth ratio is assumed, and the dimensions of the section are calculated from the maximum permissible stresses of the materials; this is therefore a direct design procedure.

Rules are given for determining the depths of the neutral axis so as to produce a section with (a) a specified percentage of tension reinforcement only; (b) a specified percentage of symmetrical reinforcement; (c) the lowest possible area of (unsymmetrical) reinforcement within the limits set by the dimensions of the cross section; and (d) the lowest possible area of reinforcement when the dimensions of the cross section are not limited.

INTRODUCTION

The problem of designing reinforced concrete sections subject to combined bending and compression offers little trouble from the theoretical standpoint; it is only when an attempt is made to apply the theory to practical examples that difficulties are encountered. Unless a form of graph or table is used it is necessary to assume a section with a certain percentage of reinforcement and find the depth of the neutral axis by the solution of the cubic equation

$$k^3 + \alpha k^2 + \beta k\, A_s + \gamma A_s = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

where k is the neutral axis depth ratio, A_s the area of reinforcement, and α, β and γ are constants. This method offers no real difficulties, but it is a tedious process to find an economical section, and several attempts may have to be made before a section is found which will keep the stresses in the steel and concrete at or just below the maximum permissible stresses.

In the method proposed here these difficulties are overcome by assuming the neutral axis depth ratio, and then solving Eq. (1) as a first degree equation

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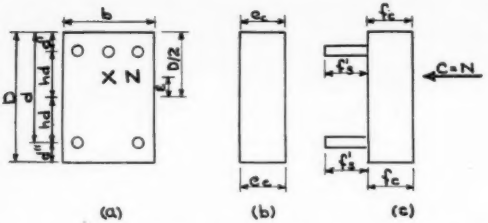

Fig. 1—Distribution of stress and strain in eccentrically loaded columns, phase 1

for A_s. For maximum efficiency the neutral axis depth ratio depends on the eccentricity of the column load and the percentage of reinforcement, and formulas are given for determining its value in terms of these quantities.

The bending moment M is defined as the moment of the column load N about the center of the tension reinforcement, and the section is designed for this moment together with the column load acting at the center of the tension reinforcement by using the principle of superposition. The dimensions of the concrete section and the reinforcement are calculated for the moment M, using the procedure for sections subject to bending only, and the area of tension reinforcement is then reduced to allow for the direct load N.

ASSUMPTIONS

The following assumptions are made in deriving the formulas: (1) plane sections remain plane and normal to the longitudinal fibers after bending, (2) both steel and concrete are elastic and obey Hooke's Law, *i.e.*, stress is proportional to strain, (3) tensile stresses in the concrete may be disregarded, (4) initial stresses in the concrete and the steel may be disregarded, (5) the bond between the concrete and the steel is sufficient to cause the materials to act as one, and (6) the centroid of the reinforced concrete section is coincident with the geometrical center of the rectangle.

ANALYSIS OF PHASES

The problem of combined bending and compression may be divided into four phases, according to the location of the neutral axis.

In the first phase the neutral axis is at infinity, and the section is uniformly strained. This phase includes concentrically loaded columns with symmetrical reinforcement, and eccentrically loaded columns with unsymmetrical reinforcement, in which the asymmetry of the reinforcement is just sufficient to balance the eccentricity of the column load (Fig. 1).

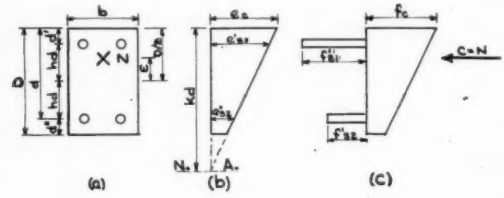

Fig. 2—Distribution of stress and strain in eccentrically loaded columns, phase 2

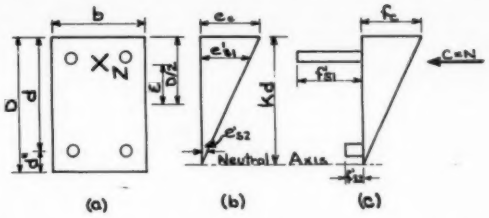

Fig. 3—Distribution of stress and strain in eccentrically loaded columns, phase 3

In the second phase the neutral axis is at a finite distance outside the section. The entire section is therefore in compression (Fig. 2).

As the neutral axis passes inside the section, there is a short transition phase in which the whole of the steel is in compression, while some of the concrete in the cover over the reinforcement is in tension. The depth of the neutral axis is then less than the over-all depth, D, of the section, but greater than the effective depth, d (Fig. 3).

In the fourth phase the depth of the neutral axis is less than the effective depth of the section, and some or all of the steel is therefore in tension (Fig. 4). The limit of this phase is reached when the column load acts at infinity, *i.e.*, the section is subject to bending only. The discussion of this paper is therefore confined to phase four.

NOTATION

a_s' = area of additional tension steel in a section with double reinforcement
A_s = area of tension reinforcement
A_s' = area of compression reinforcement
b = width of section, *i.e.*, length of rectangle parallel to the plane of bending
C = resultant compressive force in the section
d = effective depth of section
d' = depth of compression reinforcement, measured from compression edge
d'' = depth of tension reinforcement, measured from tension edge
D = over-all depth of section, *i.e.*, length of rectangle perpendicular to the plane of bending
e_c = maximum compressive strain in concrete
e_s = tensile strain at center of tension reinforcement
E_c = modulus of elasticity of concrete = $1000\ F_c'$
E_s = modulus of elasticity of steel
Σ = eccentricity of column load
f_c = actual maximum compressive stress in concrete
f_s = actual stress in tension reinforcement
f_s' = actual stress in compression reinforcement
F_c = maximum permissible concrete stress = $0.45\ F_c'$
F_c' = cylinder crushing strength of concrete
F_s = maximum permissible steel stress
g = cover ratio = d'/d or d''/d
h = ratio of distance of reinforcement from centroid of section to effective depth of section, *i.e.*, $h\ d = (D - d' - d'')/2$
j = moment arm coefficient
k = neutral axis depth ratio
K = design coefficient = $\frac{1}{2}\ F_c\ k\ j$
M = bending moment, referred to center of tension reinforcement = $N(\epsilon + h\ d)$
n = modular ratio = $E_s/E_c = 30{,}000/F_c'$
N = column load
p = tensile steel ratio = $A_s/b\ d$
p' = compressive steel ratio = $A_s'/b\ d$
p_o = balanced steel ratio for rectangular section subjected to bending only = $F_c\ k/2\ F_s$
p_{or} = balanced steel ratio for rectangular section subjected to combined bending and compression
T = resultant tensile force in section

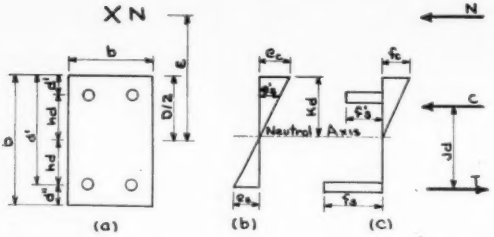

Fig. 4—Distribution of stress and strain in eccentrically loaded columns, phase 4

BALANCED DESIGN OF SECTIONS WITH TENSION REINFORCEMENT ONLY

If the eccentricity of the column load is large, the most economical solution is a section with tension reinforcement only, in which both the steel and the concrete are stressed to the full value of the maximum permissible stresses, *i.e.*, $f_c = F_c$ and $f_s = F_s$. This is called a *balanced design.*

Since it is assumed that plane sections remain plane, the variation of strain throughout the section is linear, as shown in Fig. 5b, and therefore

$$e_c/e_s = kd/(d - kd)$$

Assuming that both steel and concrete obey Hooke's Law

$$\frac{e_c}{e_s} = \frac{f_c}{f_s} \cdot \frac{E_s}{E_c} = n\frac{f_c}{f_s} = n\frac{F_c}{F_s}$$

Therefore

$$kd/(d - kd) = n\,F_c/F_s$$

and the neutral axis depth ratio

$$k = n\,F_c/(n\,F_c + F_s) \quad \ldots\ldots(2)$$

This equation is the same as for a rectangular section subject to bending only (Fig. 6).

The resultant compressive force in the section acts at the center of gravity of the triangle (Fig. 5c), and therefore the moment arm coefficient

$$j = 1 - k/3 \quad \ldots\ldots(3)$$

The resultant compressive force in the section

$$C = \tfrac{1}{2}\,F_c\,b\,kd \quad \ldots\ldots(4)$$

and the resultant tensile force

$$T = F_s\,A_s = F_s\,p_{oc}\,b\,d \quad \ldots\ldots(5)$$

where p_{oc} is the balanced steel ratio for a section subject to combined bending and compression.

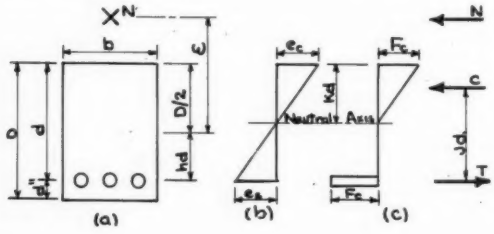

Fig. 5—Distribution of stress and strain in eccentrically loaded columns with tension reinforcement only, phase 4

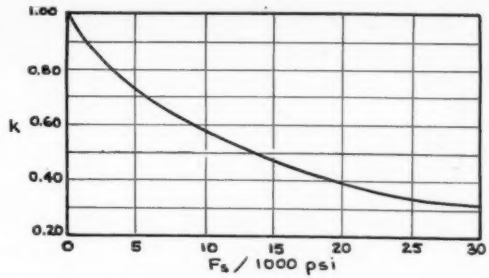

Fig. 6—Neutral axis depth ratio, k. (Since $nF_c = 13{,}500$ psi, irrespective of the strength of the concrete, the value of k varies only with the steel stress, f_s.)

For vertical equilibrium the column load

$$N = C - T$$

and therefore, substituting from Eq. (4) and (5).

$$p_{oc} = \frac{F_c}{2\,F_s} \cdot k - \frac{N}{F_s\,b\,d} = p_o - \frac{N}{F_s\,b\,d} \quad \ldots\ldots (6)$$

where $p_o = F_c\,k/2\,F_s$ is the balanced steel ratio for a rectangular section subject to bending only, *i.e.*, for $N = 0$ (Fig. 7).

Taking moments about the center of the tension reinforcement

$$\begin{aligned} M &= N\,(\epsilon + hd) = N\,(\epsilon + d/2 - d''/2) \\ &= N\,[\epsilon + d/2\,(1 - g)\,] \\ &= \tfrac{1}{2}\,F_c\,b\,kd\,jd = K\,bd^2 \end{aligned} \quad \ldots\ldots (7)$$

where $K = \frac{1}{2}\,F_c\,k\,j$ is the design coefficient for the balanced design of a rectangular section subject to bending only (Fig. 8). Therefore the effective depth required for the eccentrically loaded column

$$d = \sqrt{M/K\,b} \quad \ldots\ldots (8)$$

and the area of reinforcement

$$A_s = p_o\,b\,d - N/F_s \quad \ldots\ldots (9)$$

For large eccentricity ratios the bending moment M may be obtained from Eq. (7) by estimating the value of hd. For eccentricity ratios of the order of unity, however, this may be considerably in error. From Eq. (7)

$$K\,bd^2 = N\epsilon + N\,d\,(1 - g)/2 \quad \ldots\ldots (10)$$

This is a quadratic equation for d:

$$d = \frac{N}{4\,K\,b}\,(1 - g) + \sqrt{\left(\frac{N}{4\,K\,b}\,(1 - g)\right)^2 + \frac{N\epsilon}{K\,b}} \quad \ldots\ldots (11)$$

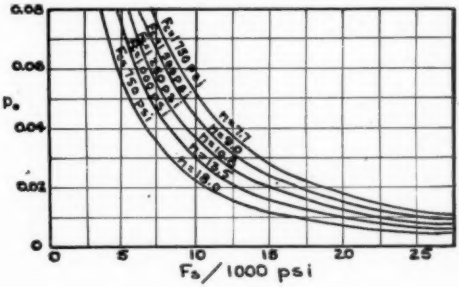

Fig. 7—Steel ratio, p_o, for the balanced design of a section with tension reinforcement only subject to pure bending

The theoretical lower limit is reached when a section without any tension reinforcement is obtained, *i.e.*,

$p_{oc} = 0 = p_o - N/F_s\, b\, d$

Substituting from Eq. (2) and (6), the theoretical maximum value of the column load is therefore

$N = p_o F_s\, b\, d = n F_c^2\, b\, d/2\,(n F_c + F_s)$

In practice it is necessary to provide a small amount of reinforcement at all times. If the minimum steel ratio is p, then the maximum value of the column load for balanced design, using tension reinforcement only, is

$$N = \left(\frac{n F_c^2}{2\,(n F_c + F_s)} - p F_s\right) b\, d \qquad (12)$$

Example 1

A rectangular column, 20 in. wide, carries an eccentric load of 25,000 lb acting at a point 40 in. from the centroid of the section, as shown in Fig. 9. Find the effective depth and the area of reinforcement required for balanced design, if the maximum permissible stresses for steel and concrete are 18,000 and 1000 psi respectively, and the modular ratio is 13.5.

The eccentricity of the column load is large, and the effective depth may therefore be obtained from Eq. (8). Assuming that the column has an over-all depth $D = 20$ in. and a cover ratio $g = 0.10$, the distance of the reinforcement from the centroid of the section

$hd = D/2 - d'' = 8$ in.

and the bending moment, referred to the center of the tension reinforcement,

$M = N\,(\epsilon + hd) = 25{,}000\,(40 + 8) = 1{,}200{,}000$ in.-lb.

With the maximum permissible stresses given, the design coefficient and the steel ratio for balanced design for a section subject to bending only are 184 psi and 0.0119 respectively (Figs. 8 and 7).

From Eq. (8) the effective depth

$d = \sqrt{1{,}200{,}000/184 \times 20} = 18.0$ in.

and the over-all depth $D = d\,(1 + g) = 19.8$ in. From Eq. (9) the area of tension reinforcement required

$A_s = 0.0119 \times 20 \times 18.0 - 25{,}000/18{,}000 = 2.90$ sq in.

The steel ratio for the section

$p = A_s/b\, d = 2.90/20 \times 18 = 0.008$

OVER-REINFORCED AND UNDER-REINFORCED SECTIONS WITH TENSION REINFORCEMENT ONLY

If the column load is greater than the value obtained from Eq. (12), the stress in the tension reinforcement, f_s, must be kept below the maximum

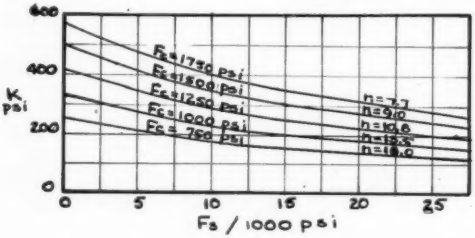

Fig. 8—Design coefficient, K, for balanced design of a section with tension reinforcement only subject to pure bending

permissible stress, F_s, so as to keep the area of reinforcement above the minimum value laid down by the Building Code. The design is then *over-reinforced.*

This process is equivalent to designing the section with maximum permissible stresses F_c and f_s for the concrete and steel respectively.

Taking moments about the center of the tension reinforcement

$$M = N\,(\epsilon + d/2 - d''/2) = K\,b\,d^2$$

where K is the design coefficient for balanced design with maximum permissible stresses F_c and f_s (Fig. 8). Substituting for K and j from Eq. (7) and (3)

$$N\,(\epsilon + d/2 - g/2) = \tfrac{1}{2}\,F_c\,k\,(1 - k/3)\,b\,d$$

and

$$\frac{N}{F_c\,b\,d} = \frac{k\,(3 - k)}{6\,(\epsilon/d + \tfrac{1}{2} - g/2)} \qquad (13)$$

The area of reinforcement required

$$A_s = p\,b\,d = p_o\,b\,d - N/f_s$$

where p_o is the steel ratio for balanced design for a section subject to bending only with maximum permissible stresses f_s and F_c (Fig. 7). Substituting for p_o from Eq. (6)

$$p\,b\,d = \frac{F_c}{2\,f_s}\,k\,b\,d - \frac{N}{f_s}$$

and therefore

$$\frac{N}{F_c\,b\,d} = \frac{k}{2} - p\,\frac{f_s}{F_c} \qquad (14)$$

From Eq. (13) and (14)

$$p = \frac{k\,F_c}{2\,f_s}\left[1 - \frac{3 - k}{3\,(\epsilon/d + \tfrac{1}{2} - g/2)}\right] \qquad (15)$$

In practice the problem reduces to selecting the appropriate value of k for a given eccentricty ratio ϵ/d and a given steel ratio p. The answer can be obtained from a series of curves such as those of Fig. 10 and 10a, or else by solving Eq. (15) as a quadratic equation for k.

If the maximum stress in the concrete is some value of f_c smaller than F_c, while the stress in the tension reinforcement is F_s, then the section is *under-reinforced.* The problem is solved in the same manner as the over-reinforced section, and the steel ratio

$$p = \frac{k\,f_c}{2\,F_s}\left[1 - \frac{3 - k}{3\,(\epsilon/d + \tfrac{1}{2} - g/2)}\right] \qquad (16)$$

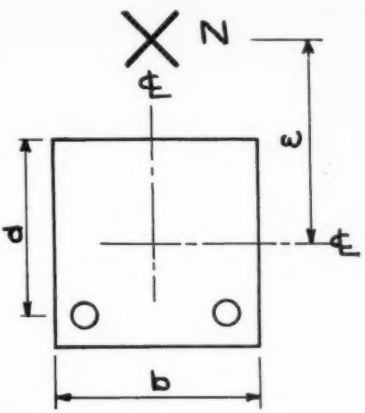

Fig. 9—Loading assumption for column in example 1

Example 2

Revise example 1 so as to increase the steel ratio to approximately 0.02.

Taking the effective depth from example 1, the ratio $d/\epsilon = 18.0 / 40 = 0.45$. From Fig. 10, k should be approximately 0.56 for 2 percent reinforcement. The steel stress is therefore limited to $nF_c\,(1 - k)/k = 10{,}600$ psi (Fig. 6).

The moment arm coefficient $j = 1 - k/3 = 0.81$. The design coefficient $K = \frac{1}{2} F_c\, k\, j = 226$ psi (Fig. 8)

and the steel ratio for pure bending

$$p_o = \frac{F_c}{2 f_s} k = 0.0264 \text{ (Fig. 7).}$$

From Eq. (8) the effective depth $d = \sqrt{1{,}200{,}000/226 \times 20} = 16.4$ in.
and the over-all depth $D = 16.4 \times 1.10 = 18.1$ in.

From Eq. (9) the area of reinforcement

$A_s = 0.0264 \times 20 \times 16.4 - 25{,}000/10{,}600 = 6.30$ sq in.
which corresponds to a steel ratio of 0.0195.

Example 3

A rectangular concrete column, 10 in. wide, carries an eccentric load of 25,000 lb, acting at a point 10 in. from the centroid of the section, as shown in Fig. 9. Find the effective depth and the area of tension steel required, using an over-reinforced design with approximately 1 percent reinforcement, if the maximum permissible stress for concrete is 1000 psi, and the modular ratio is 13.5.

Assuming an effective depth of approximately 14 in., the ratio $d/\epsilon = 1.40$. From Fig. 10, 1 percent reinforcement corresponds to a k value of 0.57. From Fig. 6, 7, and 8 $f_s = 10{,}100$ psi; $p_o = 0.0295$; and $K = 230$ psi.

Since the eccentricity of the column load is relatively small, the effective depth cannot be obtained with sufficient accuracy from Eq. (8), and Eq. (11) is used instead. Assuming a cover ratio $g = 0.10$

$$d = \frac{25{,}000\,(1 - 0.1)}{4 \times 230 \times 10} + \sqrt{\left(\frac{25{,}000\,(1 - 0.1)}{4 \times 230 \times 10}\right)^2 + \frac{25{,}000 \times 10}{230 \times 10}} = 13.2 \text{ in.}$$

From Eq. (9) the area of reinforcement $A_s = 0.0295 \times 10 \times 13.2 - 25{,}000/10{,}100 = 1.26$ sq in. which corresponds to a steel ratio of 0.0096.

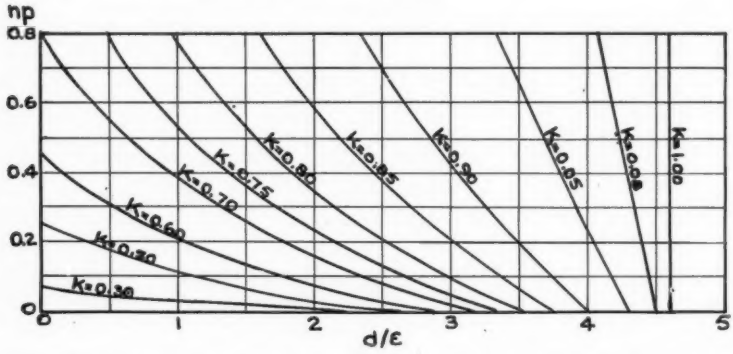

Fig. 10—Neutral axis depth ratio for over-reinforced sections with tension reinforcement only. Cover ratio, $g = 0.10$

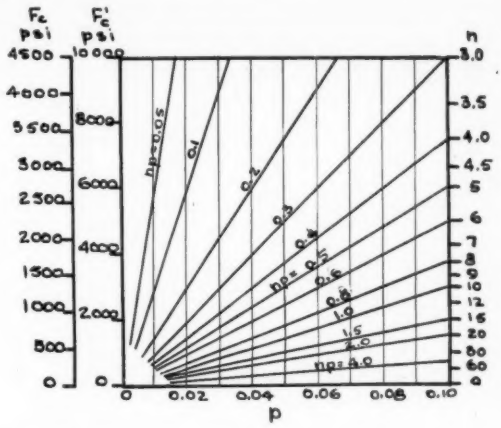

Fig. 10a—Relation between values of np and p for various concrete strengths

BALANCED DESIGN OF SECTIONS WITH TENSION AND COMPRESSION REINFORCEMENT

The solution of this problem is easily obtained by making use of the principle of superposition. The eccentric load N is replaced by a load N acting at the center of the tension reinforcement plus a moment

$$M = N(\epsilon + hd) = N(\epsilon + d/2 - d''/2) \quad \ldots(17)$$

where ϵ is the eccentricity of the load N measured from the centroid of the section, and hd is the distance of the center of the tension reinforcement from the centroid of the section (Fig. 11). The centroid of the section is assumed to be coincident with the geometrical center of the rectangle.

The design is carried out in three stages (shown diagrammatically in Fig. 12). The resistance moment of a balanced section with tension reinforcement only is first calculated. Additional reinforcement is then provided to take the balance of bending moment. Finally the area of tension reinforcement is reduced to allow for the direct compression of the column load acting at the center of the tension reinforcement.

For balanced design the moment

$$M_1 = K\,b\,d^2 \quad \ldots(18)$$

and the area of reinforcement

$$M_1/F_s\,jd = p_o\,b\,d \quad \ldots(19)$$

where j is the moment arm coefficient for balanced design. The remainder

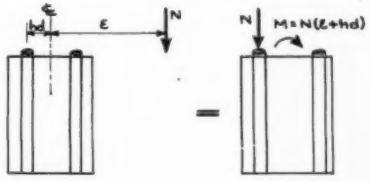

Fig. 11—Definition of bending moment, M, in terms of eccentricity of direct load, N

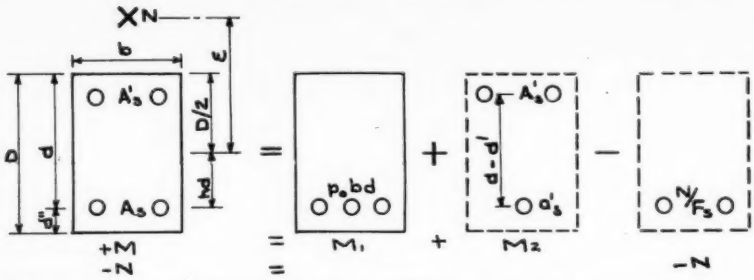

Fig. 12—Principle of superposition applied to sections with tension and compression reinforcement

of the resistance moment is provided by additional tension and compression reinforcement.

$$M_2 = M - M_1 \quad (20)$$

Neglecting the area of concrete displaced by the compression reinforcement, the resistance moment of the additional steel is

$$M_2 = F_s a_s' (d - d') = f_s' A_s' (d - d') \quad (21)$$

and therefore

$$a_s' = M_2/F_s (d - d') \quad (22)$$

In balanced design the tension steel is stressed to its maximum permissible stress, F_s. The stress in the compression reinforcement is therefore (Fig. 4)

$$f_s' = \frac{kd - d'}{d - kd} \cdot F_s \quad (23)$$

where the neutral axis depth ratio for balanced design

$$k = n F_c/(n F_c + F_s)$$

The area of compression reinforcement required is therefore

$$A_s' = a_s' F_s/f_s' = a_s' \frac{d - kd}{kd - d'} = a_s' \frac{1 - k}{k - g} \quad (24)$$

The direct compressive load N acting at the center of the tension reinforcement reduces the area of tension steel required by N/F_s, and the total area of tension reinforcement is therefore

$$A_s = p_o b d + a_s' - N/F_s \quad (25)$$

Example 4

Using a balanced design, find the area of tension and compression reinforcement required for column carrying the same load as in example 2, and having the same overall dimensions.

N = 25,000 lb.; e = 40 in.; b = 20 in.; D = 18.1 in.; d = 16.4 in.; g = 0.10; F_c = 1000 psi; F_s = 18,000 psi; n = 13.5.

For balanced design k = 0.429; K = 184 psi; and p_o = 0.0119 (Fig. 6, 7, and 8). The distance of the tension reinforcement from the centroid kd = (1 − 0.10) 16.4 / 2 = 7.38 in., and the bending moment, referred to the center of the tension reinforcement,

$$M = 25{,}000\ (40 + 7.38) = 1{,}185{,}000 \text{ in.-lb.}$$

The portion of this moment carried by the concrete

$$M_1 = 184 \times 20 \times 16.4^2 = 990{,}000 \text{ in.-lb}$$

and the area of tension reinforcement required by the moment M_1 for balanced design

$$p_o b d = 0.0119 \times 20 \times 16.4 = 3.90 \text{ sq in.}$$

The remainder of the bending moment

$$M_2 = 1{,}185{,}000 - 990{,}000 = 195{,}000 \text{ in.-lb.}$$

The area of tension reinforcement required by moment M_2

$$a_s' = 195{,}000/18{,}000 \times 16.4\ (1 - 0.10) = 0.73 \text{ sq in.}$$

and the area of compression reinforcement

$$A_s' = 0.73\,\frac{1 - 0.429}{0.429 - 0.100} = 1.31 \text{ sq in.}$$

(This is also the net area of compression reinforcement.)

The direct load reduces the area of tension reinforcement by $N/F_s = 25{,}000/18{,}000 = 1.39$ sq in., and the net area of reinforcement is therefore

$$A_s = 3.90 + 0.73 - 1.39 = 3.24 \text{ sq in.}$$

The total area of reinforcement $A_s + A_s' = 3.24 + 1.31 = 4.55$ sq in. which is 28 percent less than the area of reinforcement calculated in example 2. The problem of design for minimum steel ratio is discussed later and reference to Fig. 16 will show that the balanced design is indeed the most economical for this section.

BALANCED DESIGN OF SECTIONS WITH SYMMETRICAL REINFORCEMENT

In balanced design the ratio A_s'/A_s is dependent on the dimensions of the section, and according to how these are chosen, it may be greater or smaller than unity. It is often an advantage to have symmetrical reinforcement. This simplifies the detailing of the column, particularly when the area of reinforcement is small. It is always the most economical arrangement if the bending moment is liable to change sign while remaining constant in magnitude. Putting $A_s' = A_s$ and substituting from Eq. (25) and (24)

$$p_o\, b\, d + a_s' - \frac{N}{F_s} = a_s'\,\frac{d - kd}{kd - d'}$$

Substituting for N from Eq. (17), for $p_o\, b\, d$ from Eq. (19), and for a_s' from Eq. (22)

$$\frac{M_1}{F_s\, jd} + \frac{M_2}{F_s\,(d - d')} - \frac{M}{F_s\left(\epsilon + \dfrac{d}{2} - \dfrac{d''}{2}\right)} = \frac{M_2}{F_s\,(d - d')} \cdot \frac{d - kd}{k\,d - d'}$$

For symmetrical reinforcement the cover ratio is the same for both faces, *i.e.*, $d' = d''$. Substituting for M^2 from Eq. (20), and dividing throughout by $F_s \times d$

$$\frac{M_1}{j} + \frac{M - M_1}{1 - g} - \frac{2\,M}{2\,\epsilon/d + 1 - g} = \frac{M - M_1}{1 - g} \cdot \frac{1 - k}{k - g}$$

and therefore

$$\frac{M_1}{M} = \frac{\dfrac{1 - 2\,k + g}{(k - g)\,(1 - g)} + \dfrac{2}{2\,\epsilon/d + 1 - g}}{\dfrac{1 - 2\,k + g}{(k - g)\,(1 - g)} + \dfrac{1}{j}} \qquad (26)$$

In Fig. 13 the ratio M_1/M is plotted against the reciprocal of the eccentricity ratio, d/ϵ, for cover ratios from 0 to 0.20. The maximum area of reinforcement is required for pure bending, when $d/\epsilon = 0$; the ratio M_1/M is then approximately $\frac{1}{2}$. For $d/\epsilon = 1$, M_1/M is approximately $\frac{3}{4}$. For $d/\epsilon = 2.5$ (approx.) M_1/M becomes unity, *i.e.*, the column is unreinforced. The

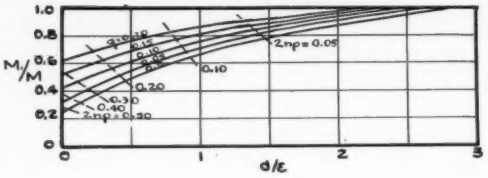

Fig. 13—M_1/M for balanced sections with symmetrical reinforcement—maximum permissible steel stress, F_s = 18,000 psi. For different maximum permissible steel stresses the values of M_1/M may be interpolated from Fig. 14

practical limit is reached when $2p = 0.01$, which is the minimum steel ratio permitted by the ACI Code. The eccentricity ratio ϵ/d is then between 0.6 and 0.8, depending on the modular ratio (Table 1).

Example 5

Redesign the column of example 1 as a balanced square section with symmetrical reinforcement.

N = 25,000 lb.; ϵ = 40 in.; $b = D = d(1 + g)$; g = 0.15; F_s = 18,000 psi; F_c = 1000 psi; n = 13.5.

For balanced design

k = 0.429; K = 184 psi; p_o = 0.0119.

Assuming a column 18 in. square, the effective depth d = 15.7 in. The distance of the tension reinforcement from the centroid of the section $hd = d/2\,(1 - g)$ = 6.7 in., and the bending moment referred to the center of the tension reinforcement, is M = 25,000 (40 + 6.7) = 1,166,000 in.-lb.

The ratio d/ϵ = 0.39. From Fig. 13 the ratio M_1/M should be taken as approximately 0.71. The portion of the bending moment taken by the concrete is therefore M_1 = 0.71 × 1,166,000 = 829,000 in.-lb and the effective depth

$$d = b/(1 + g) = \sqrt[3]{M_1/K\,(1 + g)} = \sqrt[3]{829{,}000/184 \times 1.15} = 15.8 \text{ in.}$$

The width and the over-all depth $b = D = 1.15\,d$ = 18.1 in., and the depth of the neutral axis kd = 0.429 × 15.8 = 5.8 in. The area of tension reinforcement required by the moment M_1 for balanced design

$$p_o\,b\,d = 0.0119 \times 18.1 \times 15.8 = 3.42 \text{ sq in.}$$

TABLE 1—CRITICAL VALUES OF ECCENTRICITY RATIO, ϵ/d, SHOWING THE RANGE OF THE FOUR PHASES FOR SECTIONS WITH SYMMETRICAL REINFORCEMENT, DISREGARDING THE TENSILE STRENGTH OF THE CONCRETE (STANDARD CLASSIFICATION)*

$2np$	g	Upper limit of phase 4	Upper limit of phase 3 Lower limit of phase 4	Upper limit of phase 2 Lower limit of phase 3	Phase 1 Lower limit of phase 2
0	0	∞	0.167	0.167	0
0.15	0.05	∞	0.220	0.202	0
0.30	0.10	∞	0.266	0.214	0
0.45	0.15	∞	0.292	0.210	0
0.60	0.20	∞	0.310	0.200	0

*The critical value of the eccentricity ratio are computed from the well-known formulas for checking symmetrical sections, as quoted in the *Reinforced Concrete Design Handbook*, American Concrete Institute, 1939. The cover ratio is increased with the steel ratio to allow for the greater thickness of cover required in heavily reinforced sections.

The remainder of the bending moment

$$M_2 = 1{,}166{,}000 - 829{,}000 = 337{,}000 \text{ in.-lb.}$$

The area of tension reinforcement required by moment M_2

$$a_s' = 337{,}000/18{,}000 \times 15.8\ (1 - 0.15) = 1.39 \text{ sq in.}$$

and the area of compression reinforcement

$$A_s' = 1.39\ \frac{1 - 0.429}{0.429 - 0.100} = 3.53 \text{ sq in.}$$

The direct load reduces the area of tension reinforcement by $N/F_s = 25{,}000/18{,}000 = 1.39$ sq in., and the net area of tension reinforcement

$$A_s = 3.42 + 1.39 - 1.39 = 3.42 \text{ sq in.}$$

Although there is a slight difference in the numerical values of A_s and A_s', this is not significant when the areas are translated into actual bar sizes, and the reinforcement is therefore in practice symmetrical. Small variations in M_1/M are negligible.

OVER-REINFORCED AND UNDER-REINFORCED SECTIONS WITH SYMMETRICAL REINFORCEMENT

From an examination of Fig. 14a it is apparent that columns with symmetrical reinforcement and medium eccentricity ratios must be *over-reinforced, i.e.*, the stress in the tension reinforcement, f_s, must be less than F_s.

The procedure is similar to that outlined for balanced sections, and the design is carried out in three stages.

$$M_1 = K\ b\ d^2 \qquad (18)$$

where K is the design coefficient for balanced design with maximum permissible stresses f_s and F_c (Fig. 8). The corresponding area of tension reinforcement

$$M_1/f_s\ jd = p_o\ b\ d \qquad (19)$$

where p_o is the steel ratio for balanced design with maximum permissible stresses f_s and F_c (Fig. 7). The remainder of the resistance moment is provided by additional tension and compression steel.

$$M_2 = M - M_1 \qquad (20)$$

The area

$$a_s' = M_2/f_s\ (d - d') \qquad (22a)$$

and the area of compression reinforcement

$$A_s' = a_s'\ \frac{d - kd}{kd - d'} = a_s'\ \frac{1 - k}{k - g} \qquad (24)$$

where $k = n\ F_c/(n\ F_c + f_s)$ (Fig. 6). The direct compression acting at the

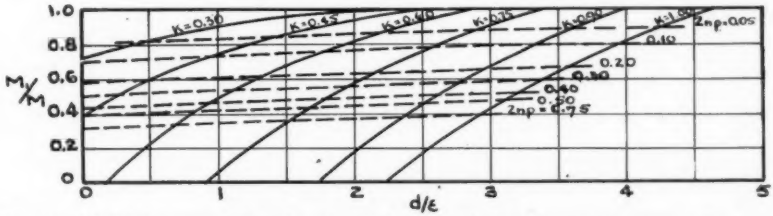

Fig. 14—M_1/M for over-reinforced section with symmetrical reinforcement—cover ratio, g = 0.10. For different cover ratios values of M_1/M may be interpolated from Fig. 13

center of the tension reinforcement reduces the tension steel by N/f_s, and the net area is therefore

$$A_s = p_o\, b\, d + a_s' - N/f_s \quad \text{(25a)}$$

If the reinforcement is to be symmetrical, the ratio M_1/M must be in accordance with Eq. (26). The ratio M_1/M is plotted against d/ϵ in Fig. 14 for k values between 1.0 and 0.3 and a cover ratio of 0.10.

If the maximum stress in the concrete is some value f_c less than F_c, while the stress in the tension reinforcement is equal to the maximum permissible stress F_s, then the section is *under-reinforced.* The problem can be solved in the same manner as that of the over-reinforced section, and the value of M_1/M appropriate to the steel ratio and eccentricity ratio of the section can be selected from Fig. 14a. Although a saving of steel can be achieved for sections with very large eccentricities through using an under-reinforced design, this economy is usually offset by the increased cross-sectional area of the column, as compared with a balanced section. The problem is therefore largely of academic interest.

The theoretical lower limit is reached when $k = 0$, *i.e.*, when the neutral axis becomes coincident with the center of the tension reinforcement. The area of tension reinforcement is unstressed and becomes indeterminate.

If the stress in the tension reinforcement is very small, the area of tension steel is the difference between two very large quantities, $p_o\, b\, d + a_s'$ and N/f_s. The accurate determination of this difference is difficult, and it is therefore convenient in practice to include these sections in phase 2. This can be justified on theoretical grounds, since the section remains homogeneous until cracks begin to form on the tension face, and its second moment of area, I, should therefore be calculated by taking account of the full cross sectional area. Limiting eccentricity ratios thus obtained are given in Table 2.

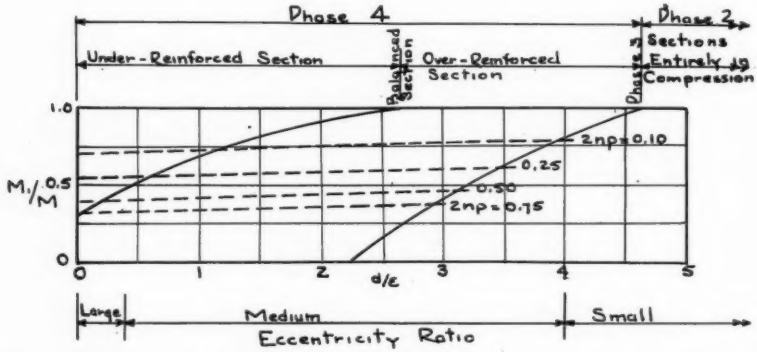

Fig. 14a—This diagram is identical with Fig. 14, all the k lines except those for balanced design and phase limits have been omitted for clarity to show that columns with medium eccentricities must be over-reinforced

TABLE 2—CRITICAL VALUES OF ECCENTRICITY RATIO, ϵ/d, FOR SECTIONS WITH SYMMETRICAL REINFORCEMENT, ALLOWING FOR THE TENSILE STRENGTH OF THE CONCRETE (MODIFIED CLASSIFICATION)*

$2np$	g	Upper limit of phase 4	Upper limit of phase 2 Lower limit of phase 4	Phase 1 Lower limit of phase 2
0	0	∞	0.226	0
0.15	0.05	∞	0.312	0
0.30	0.10	∞	0.372	0
0.45	0.15	∞	0.410	0
0.60	0.20	∞	0.440	0

*The critical values of the eccentricity ratio are computed in the same manner as those given in Table 1, except that the section is assumed to remain homogeneous until the *ultimate* tensile stress is reached at the tension face of the column. On this assumption the lower range of phase 4 and the whole of phase 3 are absorbed in phase 2. For computing this table the ultimate tensile strength of concrete is taken as one-tenth of the cylinder crushing strength.

Example 6

A square concrete column with approximately 3 percent of symmetrical reinforcement carries a load of 25,000 lb acting at a point 5 in. from the centroid of the section as shown in Fig. 15. Calculate the dimensions of the section if the maximum permissible stresses for steel and concrete are 18,000 and 1000 psi respectively, the modular ratio is 13.5.

$N = 25{,}000$ lb; $\epsilon = 5$ in.; $b = D$; $A_s = A_s'$ (approx.); $2p = 0.03$ (approx.). Assuming a cover ratio $g = 0.10$, and an effective depth of 8 in., the ratio d/ϵ is 1.60. From Fig. 14 it is evident that the section must be over-reinforced; for $2np = 0.405$ the ratio M_1/M should be taken as 0.61 and the value of k as 0.64.

(It will be noted that the value of M_1/M in Fig. 14 is virtually independent of the ratio d/ϵ, and it can therefore be read off Fig. 13 for cover ratios other than 0.10. A small error in the value of either k or M_1/M makes only a negligible difference in the dimensions of the section.) The stress in the tension reinforcement is therefore limited to $f_s = 1000 \times 13.5 \times 0.36/0.64 = 7600$ psi, and the design constants are $p_o = 0.042$ and $K = 252$ psi (Fig. 7 and 8).

The distance of the tension reinforcement from the centroid of the section is $hd = \frac{1}{2}\, d\,(1 - g) = 3.6$ in., and the bending moment referred to the center of the tension reinforcement is therefore

$$M = 25{,}000\ (5 + 3.6) = 215{,}000 \text{ in.-lb.}$$

The portion of this moment carried by the concrete

$$M_1 = 0.61 \times 215{,}000 = 131{,}000 \text{ in.-lb}$$

and the effective depth

$$d = \sqrt[3]{131{,}000/252 \times 1.10} = 7.7 \text{ in.}$$

The width and the over-all depth $b = D = 7.7 \times 1.10 = 8.5$ in. The area of the tension reinforcement required by the moment M_1

$$p_o\, b\, d = 0.042 \times 8.5 \times 7.7 = 2.76 \text{ sq in.}$$

The remainder of the bending moment

$$M_2 = 215{,}000 - 131{,}000 = 84{,}000 \text{ in.-lb.}$$

The area of tension reinforcement required by the moment M_2

$$a_s' = 84{,}000/7{,}600 \times 7.7\ (1 - 0.10) = 1.59 \text{ sq in.}$$

and the area of compression reinforcement

$$A_s' = 1.59\, \frac{1 - 0.64}{0.64 - 0.10} = 1.06 \text{ sq in.}$$

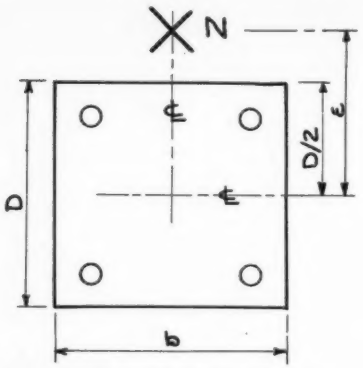

Fig. 15—Loading condition for column in example 6

The direct load reduces the area of tension reinforcement by $N/f_s = 25{,}000/7600 = 3.29$ sq in., and the net area of tension reinforcement is therefore

$A_s = 2.76 + 1.59 - 3.29 = 1.06$ sq in. The total area of reinforcement $A_s + A_s' = 2.12$ sq in., and the total steel ratio is $2.12/8.5 \times 7.7 = 0.032$.

DESIGN OF SECTIONS WITH TENSION AND COMPRESSION REINFORCEMENT FOR MAXIMUM STEEL ECONOMY

If the dimensions of the column are not specified, the condition of maximum steel economy is achieved by using a section with tension reinforcement only. Where this leads to an excessively deep section the effective depth may have to be limited to a smaller figure, and compression reinforcement must then be used. Symmetrical tension and compression reinforcement is likely to be economical for small columns, since it reduces the cost of fabrication; for large columns and arches, however, the most important consideration is a reduction in the total area of reinforcement to the minimum which can be achieved within the limits set by the over-all dimensions of the section.

The neutral axis depth ratio k for which the total steel ratio is a minimum, the eccentricity ratio ϵ/d and the ratio $N/F_c\, b\, d$ are related by the equation

$$\frac{F_c\, b\, d}{N} = 3\,\frac{g\,(1-k)^2 + (k-g)^2 - 2\,(k^2-g)\,\epsilon/d}{k\,[(k-g)\,(k\,(1-k) + 3\,(2-k)\,(k-g)\,) + (3-k)\,(g\,(1-k) - (k-g)\,)\,]} \quad (27)$$

This is a straight line relationship between $F_c\, b\, d\, /\, N$ and ϵ/d for any given value of g and k, and the families of straight lines are drawn in Fig. 16, 17, and 18 for three different cover ratios. Eq. (27) is derived in Appendix 1.

It should be noted that the steel stress f_s cannot exceed the maximum permissible stress F_s. The neutral axis depth ratio cannot therefore be decreased below the figure required for balanced design without reducing the concrete stress below the maximum permissible stress, which in turn reduces the stress in the compression reinforcement. For columns with large eccentricities the balanced design consequently represents the most economical solution.

If the eccentricity is liable to change from $+\,\epsilon$ to $-\,\epsilon$, the most economical section is evidently a column with symmetrical reinforcement, and Eq. (26) becomes the criterion of maximum steel economy.

Example 7

Using the same over-all dimensions, revise the design of example 6 to reduce the total area of steel to a minimum.

$N = 25{,}000$ lb; $\epsilon = 5$ in.; $b = D = 8.5$ in.; $d = 7.7$ in.; $g = 0.10$; $F_s = 18{,}000$ psi; $F_c = 1000$ psi; $n = 13.5$.

The ratios $\epsilon/d = 0.65$ and $F_c\, b\, d/N = 2.62$. From Fig. 17, the neutral axis depth ratio should be approximately 0.52 for maximum steel economy. (A small error in the

value of k makes only a negligible difference to the *total* area of reinforcement). The corresponding tensile stress in the reinforcement

$$f_s = 13.5 \times 1{,}000 \times 0.48/0.52 = 12{,}500 \text{ psi}$$

From Figs. 7 and 8

$$p_o = 0.0204 \quad \text{and} \quad K = 216 \text{ psi}$$

The bending moment, referred to the center of the tension reinforcement

$$M = N(\epsilon + hd) = 215{,}000 \text{ in.-lb}$$

and the portion of this moment carried by the concrete

$$M_1 = 216 \times 8.5 \times 7.7^2 = 109{,}000 \text{ in.-lb.}$$

The area of tension reinforcement required by the moment M_1

$$p_o\, b\, d = 0.0204 \times 8.5 \times 7.7 = 1.34 \text{ sq in.}$$

The remainder of the bending moment

$$M_2 = 215{,}000 - 109{,}000 = 106{,}000 \text{ in.-lb.}$$

The area of tension reinforcement required by this moment

$$a_s' = 106{,}000/12{,}500 \times 7.7\,(1 - 0.10) = 1.22 \text{ sq in.}$$

and the area of compression reinforcement

$$A_s' = 1.22 \times \frac{1 - 0.52}{0.52 - 0.10} = 1.39 \text{ sq in.}$$

The direct load reduces the area of tension reinforcement by $N/f_s = 25{,}000/12{,}500 = 2.00$ sq in., and the net area of tension reinforcement is therefore

$$A_s = 1.34 + 1.22 - 2.00 = 0.56 \text{ sq in.}$$

The total area of steel $A_s + A_s' = 1.95$ sq in., which is 8 percent less than the area calculated in example 6.

For comparison the same example is repeated for balanced design.

$N = 25{,}000$ lb; $\epsilon = 5$ in., $b = D = 8.5$ in.; $d = 7.7$ in.; $g = 0.10$; $M = 215{,}000$ in.-lb.; $F_s = 18{,}000$ psi; $F_c = 1000$ psi; $n = 13.5$.

For balanced design the constants $k = 0.429$; $K = 184$ psi; and $p_o = 0.0119$. The portion of the moment carried by the concrete

$$M_1 = 184 \times 8.5 \times 7.7^2 = 93{,}000 \text{ in.-lb}$$

and the area of reinforcement required by that moment

$$p_o\, b\, d = 0.0119 \times 8.5 \times 7.7 = 0.78 \text{ sq in.}$$

The remainder of the bending moment $M_2 = 122{,}000$ in.-lb, and the corresponding areas of tension and compression reinforcement

$$a_s' = 122{,}000/18{,}000 \times 7.7 \times 0.90 = 0.98 \text{ sq in.}$$

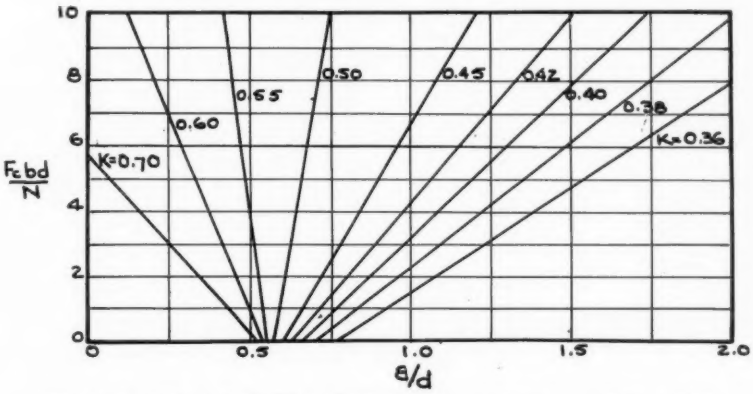

Fig. 16—Neutral axis depth ratio for maximum steel economy—cover ratio, $g = 0.05$

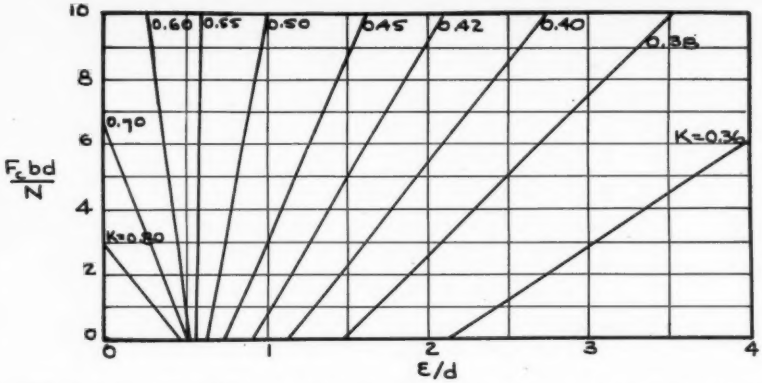

Fig. 17—Neutral axis depth ratio for maximum steel economy—cover ratio, $g = 0.10$

and $A_s' = 0.98 \times 0.571/0.329 = 1.69$ sq in.

The direct load reduces the area of tension reinforcement by $25{,}000/18{,}000 = 1.39$ sq in., and the net area of tension reinforcement is therefore $A_s = 0.78 + 0.98 - 1.39 = 0.37$ sq in.

The total area of steel $A_s + A_s' = 0.37 + 1.69 = 2.06$ sq in., which is 6 percent more than the result obtained when the section is designed for maximum steel economy.

CONCLUSION AND SUMMARY

The design of eccentrically loaded columns can be reduced to the same fundamental principles as the design of rectangular sections subject to pure bending by use of superposition. The eccentric load N is replaced by a load of equal magnitude acting at the center of the tension reinforcement together with a moment $M = N\ (e + hd)$, where $e + hd$ is the eccentricity of the column load referred to the center of the tension reinforcement.

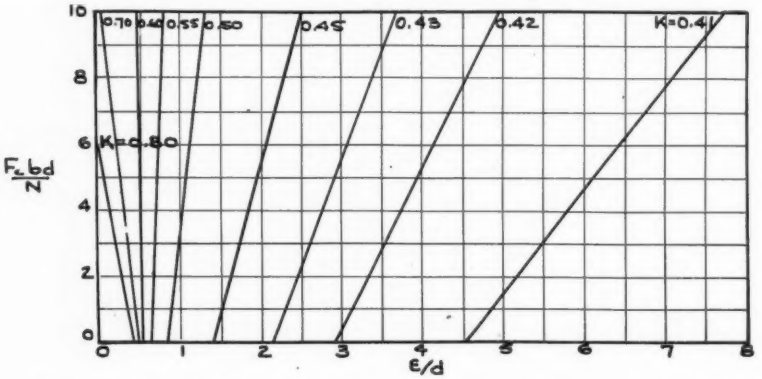

Fig. 18—Neutral axis depth ratio for maximum steel economy—cover ratio $g = 0.15$

The width b and the effective depth d of the section with tension reinforcement only are then given by the well-known equation

$$M = K\,b\,d^2$$

as in pure bending. The area of tension reinforcement

$$A_s = p_o\,b\,d - N/f_s$$

where $p_o\,b\,d$ is the area of reinforcement required in pure bending, and N/f_s is the reduction in the area of the tension reinforcement due to the direct compressive force.

For balanced design, using a 3750 psi concrete with structural grade steel reinforcement, the steel forms approximately 2 percent of the cross-sectional area when the eccentricity of the column load is large. As the eccentricity is reduced, the steel ratio decreases rapidly, the columns with medium eccentricity must therefore be *over-reinforced* so as to provide a minimum of tension steel. The limiting condition is given by Eq. (16), and the steel stresses appropriate to any given steel ratio and eccentricity ratio are plotted in Fig. 10.

The principle of superposition is also used for the design of sections with double reinforcement. The moment M is replaced by $M_1 + M_2$, where $M_1 = K\,b\,d^2$ represents the moment of resistance of the concrete with an area of tension steel reinforcement of $p_o\,b\,d$, and M_2 the moment of resistance of the additional tension reinforcement a_s' and the compression reinforcement A_s' (Fig. 12). The direct load reduces the area of tension steel by N/f_s, and the net area of reinforcement is therefore

$$A_s = p_o\,b\,d + a_s' - N/f_s$$

The ratio of tension to compression reinforcement depends on the ratio $M_1{:}M$. The case of the column with symmetrical reinforcement is of special interest, and the solution is given by Eq. (26) (Fig. 13 and 14).

The method proposed in this paper makes use only of design data required for section subject to pure bending, *i.e.*,

$$k = n\,f_c/(n\,f_c + f_s);\quad K = \tfrac{1}{2}\,f_c\,k\,j;\quad \text{and } p_o = f_c\,k/2\,f_s \text{ (Fig. 6, 7, and 8).}$$

Although this paper is intended primarily to show how the design of eccentrically loaded columns can be freed from the limitation imposed by the design charts of the standard method, a few lines on steel economy are pertinent. If the dimensions of the section are not limited, the column using the least quantity of steel is a section with tension reinforcement only. In columns with very large eccentricities of loading the steel ratio can be reduced further by using an under-reinforced design. In columns with medium eccentricities it becomes theoretically possible to eliminate reinforcement entirely, and the minimum area of steel is determined by the minimum steel ratio which is admissible in practice.

If both width and effective depth are limited, the neutral axis depth ratio for maximum steel economy is given by Eq. (27) and Fig. 16, 17, and 18.

APPENDIX 1—DERIVATION OF EQUATION (27)

From Eq. (25a) and (24)

$$A_s + A_s' = p_o\, b\, d + a_s' - N/f_s + a_s'\,(d-kd)/(kd-d')$$
$$= a_s'\,(d-d')/(kd-d') + p_o\, b\, d - N/f_s$$

Substituting $a_s' = M_2/f_s\,(d-d')$ and $p_o = F_c\, k/2\, f_s$

$$A_s + A_s' = M_2/f_s\,(kd-d') + F_c\, k\, b\, d/2\, f_s - N/f_s \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (28)$$

From Eq. (17) and (18)

$$M_2 = M - M_1 = N\,(\epsilon + d/2 - d''/2) - K\, b\, d^2$$
$$= \tfrac{1}{2}\, N\, d\,(2\,\epsilon/d + 1 - g) - \tfrac{1}{2}\, F_c\, k\,(1 - \tfrac{1}{3}k)\, b\, d^2$$

Substituting this expression into Eq. (28)

$$A_s + A_s' = \frac{N\,(2\,\epsilon/d + 1 - g)}{2\, f_s\,(k-g)} - \frac{F_c}{f_s}\cdot\frac{k(3-k)}{6(k-g)}\, b\, d + \tfrac{1}{2}\,\frac{F_c}{f_s}\, kbd - \frac{N}{f_s} \quad \ldots\ldots\ldots\ldots\ldots (29)$$

Dividing throughout by $F_c\, b\, d/6\, f_s$

$$\frac{6\,(A_s + A_s')\, f_s}{F_c\, b\, d} = \frac{3\, N\,(2\,\epsilon/d + 1 - g)}{F_c\, b\, d\,(k-g)} - \frac{k\,(3-k)}{(k-g)} + 3\,k - \frac{6\,N}{F_c\, b\, d}$$

Differentiating with respect to k

$$\frac{6\, f_s}{F_c b\, d}\cdot\frac{\partial\,(A_s + A_s')}{\partial\, k} - \frac{6\,(A_s + A_s')}{F_c\, b\, d}\cdot\frac{\partial\,(n\, F_c\,(1-k)/k)}{\partial\, k} =$$
$$-\frac{3\, N\,(2\,\epsilon/d + 1 - g)}{F_c\, b\, d\,(k-g)^2} - \frac{(k-g)\ [\,(3-k) - k\,] - k(3-k)}{(k-g)^2} + 3 - 0$$

To obtain the value of k for which $(A_s + A_s')$ is a minimum, $\partial\,(A_s + A_s')/\partial\, k = 0$.

Substituting for $A_s + A_s'$ from Eq. (29), and substituting also $f_s = n\, F_c\,(1-k)/k$ and $n\, F_c/f_s\, k^2 = 1/k(1-k)$ it follows that

$$-\frac{3\, N\,(2\,\epsilon/d + 1 - g)}{F_c\, bdk\,(1-k)\,(k-g)} + \frac{3-k}{(1-k)\,(k-g)} - \frac{3}{1-k} + \frac{6\,N}{F_c\, bdk\,(1-k)} =$$
$$\frac{3\, N\,(2\,\epsilon/d + 1 - g)}{F_c\, b\, d\,(k-g)^2} + \frac{[k\,(k-g) + g(3-k)]}{(k-g)^2} + 3$$

Multiplying throughout by $k\,(1-k)\,(k-g)^2$ and re-arranging

$$\frac{3\, N}{F_c\, bd}\,(2\,\epsilon/d + 1 - g)\,[k(1-k) - (k-g)] + \frac{6\, N}{F_c\, bd}\,(k-g)^2 =$$
$$3k\,(k-g)^2\,(1-k+1) + k^2\,(1-k)\,(k-g) + kg\,(1-k)\,(3-k) - k(3-k)\,(k-g)$$

i.e.,

$$-\frac{6\, N\,\epsilon/d}{F_c\, b\, d}\,(k^2-g) + \frac{3\, N}{F_c\, bd}\,(1-k)\,(k-g^2) + \frac{3\, N}{F_c\, bd}\,(k-g)^2$$
$$+\frac{3\, N}{F_c\, bd}\,(k-g)^2 = 3k(2-k)\,(k-g)^2 + k^2\,(1-k)\,(k-g)$$
$$+ kg\,(1-k)\,(3-k) - k\,(3-k)\,(k-g)$$

i.e.,

$$-\frac{6\, N\,\epsilon/d}{F_c\, b\, d}\,(k^2 - g) + \frac{3\, N}{F_c\, bd}\,(g - g^2 - k^2 + gk^2 + k^2 - 2gk + g^2)$$
$$+\frac{3\, N}{F_c\, bd}\,(k-g)^2 = k\,[\,(k-g)\,(3\,(2-k)\,(k-g) + k(1-k)\,)$$
$$+ (3-k)\,(g\,(1-k) - (k-g)\,)\,]$$

which gives

$$\frac{F_c\, b\, d}{N} = 3\,\frac{g\,(1-k)^2 + (k-g)^2 - 2\,(k^2-g)\,\epsilon\,/\,d}{k\,[\,(k-g)\,(k(1-k) + 3(2-k)\,(k-g)\,) + (3-k)\,(g(1-k) - (k-g)\,)\,]} \quad \ldots\ldots (27)$$

Thermal Expansion of Aggregates and Concrete Durability*

By EDWIN J. CALLAN†

SYNOPSIS

Differences in durability of concretes containing aggregates from the same source and similar concretes containing different fine and coarse aggregates are explained partially by differences in thermal expansion of the coarse aggregate and the mortar. Methods were developed to obtain simply the thermal coefficients for numerous aggregates. Concretes were tested in accelerated freezing and thawing, yielding durability factors, *DFE*, for each combination. The *DFE*s were statistically analyzed with the difference between the thermal coefficients of coarse aggregates and mortars, Δc, and coarse aggregate absorption, A, as variables. For 78 concrete combinations a relation $DFE = 109.65 - 8.76\,\Delta c - 15.22\,A$ was developed with a correlation coefficient of 0.719, which is highly significant. Thus, the durability of these concretes was reduced when the differential expansion of mortar and coarse aggregate increased. Stresses set up by such differential expansion and their effects on concrete durability are discussed briefly. It is concluded that thermal effects of this type should be considered in choosing aggregates for highly durable concretes. The methods developed for determining coefficients of thermal expansion of coarse aggregate and mortar are described.

INTRODUCTION

Concrete durability factors

Effects of thermal stresses have long been considered in the design of large concrete structures; usually based, however, on the properties of the concrete as a whole rather than on variations in thermal properties of its components. A few investigators such as Pearson,[1,3] Meissner[2] and Hornibrook[2] have reported failures in concrete ascribed to unusually low coefficients of thermal expansion of the coarse aggregates, or to aggregate having a high degree of anisotropy with respect to thermal expansion. Except for these few references, little has been published on the effects of differential thermal expansion of the coarse aggregate and mortar on the durability of concrete.

Concrete containing limestone coarse aggregates frequently has shown a high degree of resistance to laboratory freezing-and-thawing when sand manufactured from the limestone was used as fine aggregate; on the other hand, combinations of the same coarse aggregate with natural sands as fine aggregate have low resistance in many cases.

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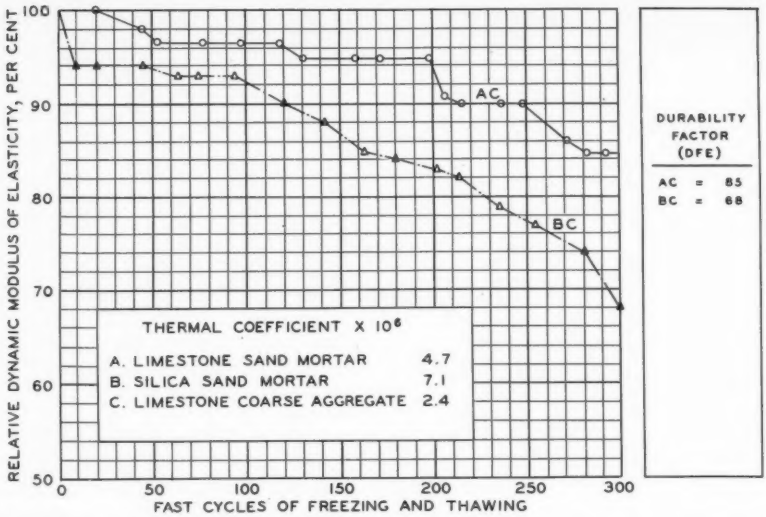

Fig. 1—Freezing and thawing test results for concrete specimens containing limestone coarse aggregate

Studies were made at the Waterways Experiment Station to determine the factors responsible for the deterioration of concretes in accelerated laboratory freezing-and-thawing. Anomalous results for a number of concretes were obtained by customary routine methods for evaluation of aggregates. The concrete mixtures conformed to the standards established for the test; and no differences in making, curing, or exposure to freezing-and-thawing were observed. Routine tests indicated that all the aggregates were independently sound and suitable for use in concrete. The slight differences in cement factors and sand-aggregate ratios for the mixtures were not sufficient to account for the large differences in durability. Fig. 1 presents typical freezing-and-thawing test results for concretes containing limestone coarse aggregate with both manufactured limestone and natural siliceous fine aggregates. A major source of difference which might account for such variation in *DFE** for similar limestone-limestone and natural sand-limestone concretes is the difference in coefficient of thermal expansion of the components of the concretes. To secure data necessary to confirm the suspicion that failure of concrete containing apparently sound materials was due to variations in thermal properties of the aggregates, several new techniques were developed for quick, accurate determination of thermal coefficients of expansion.

The role of differential thermal expansion, *i. e.*, the difference in expansivities between mortar and coarse aggregate, in the durability of concrete is discussed

*The *DFE* is the dynamic modulus of elasticity relative to the initial modulus, both determined by sonic methods, of concrete after 300 cycles of accelerated freezing and thawing if above 50 percent, or $50N/300$ where N is the number of cycles of freezing and thawing at which the concrete reached 50 percent of the initial modulus if less than 300.

from the viewpoint of producing high stresses at aggregate-mortar interfaces, causing cracking which leads to deterioration of concretes exposed to weathering influences. Rhoades and Mielenz[4] reached similar conclusions although primarily concerned with differences in coefficients of expansion between cement and aggregates, rather than between mortar and coarse aggregates. To assess this influence quantitatively, the effects of these thermal coefficient differences on the resistance of concretes to accelerated freezing and thawing were analyzed statistically.

Thermal coefficient of expansion

The determination of thermal coefficients of expansion of coarse aggregates is normally rather complicated and time consuming. The usual methods are to obtain the coefficient interferometrically, or by direct measurements using a comparator. In the interferometric method, used by Parsons and Johnson,[5] a piece of aggregate is cut into a complex shape, placed between optical prisms, and interference fringes of the pattern formed at different temperatures are counted. Direct measurements are also dependent on precise techniques, frequently requiring sensitive measuring arms and optical levers such as used by Willis and De Reus.[6] These methods yield precise results, but are time consuming and permit testing only a few specimens at a time.

A more rapid method for determining the thermal coefficients of expansion of coarse aggregate was devised and used in this investigation. Its advantages are the large number of determinations which may be made simultaneously, the relatively simple preparation of the specimens, the degree of accuracy (sufficient for all practical purposes) and applicability to many rocks which because of crumbly or fractured structure are difficult to handle by other means. The method is an application of the familiar SR-4 strain gage technique used successfully in mechanical strain determinations. In these tests the strains determined are those due to thermal expansion of unrestrained specimens over a desired temperature range.

Determination of thermal coefficient of expansion

Resistance-wire strain gages are attached to slabs of rock, the specimens are placed in a controlled-temperature cabinet, and strain readings are taken at two temperatures with an SR-4 indicator. A compensating gage and an active gage are attached to a specimen of known thermal coefficient of expansion. The readings of these two gages are applied as a correction to the readings of the gages attached to the test specimens to compensate for variations in ambient temperature, and changes in the resistance of the lead wires from this and other causes. The thermal coefficients of expansion are then computed from the corrected readings and averaged for a set of readings on the same specimen. The procedure is completely described in Appendix 1.

In addition to determining thermal coefficients of expansion of coarse aggregates, coefficients had to be secured for the mortars used in the concretes. A simple method, consisting essentially of determining length changes

of mortar bars caused by temperature changes, was devised. Standard 1 x 1 x 11½-in. bars of a standard mortar mixture were made using the various fine aggregates. After curing they were alternately immersed in water baths at different temperatures. Length readings were taken at equilibrium conditions, and coefficients of expansion computed from the length changes. The adopted procedure is presented in Appendix 2.

Following the development of these techniques, coefficients of thermal expansion were determined for a number of aggregates and mortars, the concrete combinations of which had been tested in accelerated laboratory freezing and thawing.

CONCRETE SPECIMENS

Aggregate combinations

For the statistical analysis, 78 concrete combinations, representing all aggregate combinations for which requisite data were available, were taken. The combinations were limited to those in which a single coarse aggregate had been tested in combination with several different fine aggregates for its resistance to accelerated freezing-and-thawing. These concretes are believed to constitute a representative cross section of what would normally be considered well-designed mixtures containing good aggregates. Aggregates studied were basalt, limestone, syenite, and quartzite, with manufactured sands of these materials, and natural siliceous and glacial sands. Table 1a lists the 78 combinations with values for DFE and Δc, where Δc is the difference between the coefficients of expansion of the coarse aggregate and of the mortar per deg F. Listed in Table 2 are the fine aggregates and their absorptions and loss in percent after five cycles of the magnesium sulfate soundness test. Similar information for the coarse aggregates is presented in Table 3.

Table 1b lists 19 aggregate combinations which are duplicates of several of those listed in Table 1a except that their curing period was 21 days instead of 9 days. Preliminary computations showed the combinations in Table 1b and the corresponding combinations in Table 1a to have almost identical behavior, statistically. Only a small difference in the correlation coefficient and no significant differences in emphasis of the effects of the longer curing period on the durabilities were developed.

Therefore, the data derived from specimens cured for 21 days were eliminated from the final analysis, as they tended to increase the correlations and thus unduly emphasize the weight given to the relations developed among the variables considered. The average coefficient of thermal expansion for each coarse aggregate (Table 3) was calculated after inspection of the values of the coefficient for each orientation indicated that none of the materials tested showed appreciable anisotropy. The mortars were treated as essentially homogeneous materials; coefficients of thermal expansion for those used are given in Table 2. The thermal coefficients of expansion were determined over the range 35–135 F, and the freezing-and-thawing tests in the range 0–40 F. Although the coefficients decrease with lowered temperature, the

TABLE 1—AGGREGATE COMBINATIONS TESTED BY FREEZING-AND-THAWING OF CONCRETE SPECIMENS

Fine aggregate*	Coarse aggregate†	Δc‡	*DFE*	Fine aggregate*	Coarse aggregate†	Δc‡	*DFE*
a—Combinations tested in concrete after 9-day curing at 70 F							
§6. Limestone	**20. Trap rock	0.8	96	2. Limestone	10. Limestone	2.1	74
8. Limestone	20. Trap rock	0.8	94	21. Siliceous	9. Limestone	3.5	74
20. Syenite	20. Trap rock	1.7	94	23. Siliceous	6. Limestone	3.9	74
7. Limestone	20. Trap rock	0.8	93	16. Limestone	10. Limestone	2.8	72
14. Limestone	7. Limestone	2.6	93	16. Limestone	14. Limestone	2.5	71
15. Limestone	18. Limestone	2.1	92	3. Limestone	1. Limestone	2.6	70
7. Limestone	8. Limestone	2.4	92	29. Siliceous	18. Limestone	4.2	70
8. Limestone	9. Limestone	2.4	92	1. Limestone	14. Limestone	1.8	69
9. Limestone	6. Limestone	2.6	90	1. Limestone	10. Limestone	2.1	69
28. Siliceous	19. Syenite	3.0	89	27. Siliceous	12. Limestone	4.1	69
12. Limestone	20. Limestone	0.9	88	29. Siliceous	11. Limestone	4.7	69
23. Siliceous	17. Limestone	3.2	88	28. Siliceous	3. Limestone	4.8	69
20. Syenite	19. Syenite	1.9	87	26. Siliceous	7. Limestone	3.9	67
12. Limestone	12. Limestone	2.4	87	26. Siliceous	16. Limestone	3.6	66
19. Glacial	7. Limestone	3.2	87	28. Siliceous	21. Limestone	1.9	65
29. Siliceous	19. Syenite	3.4	87	29. Siliceous	12. Limestone	4.7	65
15. Limestone	20. Trap rock	1.1	86	2. Limestone	14. Limestone	1.8	64
17. Limestone	17. Limestone	2.6	86	24. Siliceous	22. Quartzite	0.7	62
6. Limestone	11. Limestone	2.3	85	18. Glacial	6. Limestone	3.4	62
10. Limestone	5. Limestone	2.7	85	18. Glacial	7. Limestone	3.2	57
22. Limestone	21. Limestone	1.0	84	21. Siliceous	11. Limestone	3.4	55
27. Siliceous	20. Trap rock	2.6	84	24. Siliceous	15. Limestone	3.5	54
18. Glacial	17. Limestone	2.7	84	18. Glacial	16. Limestone	2.9	53
10. Limestone	20. Trap rock	0.9	83	21. Siliceous	5. Limestone	3.7	53
11. Limestone	13. Limestone	2.2	83	28. Siliceous	13. Limestone	4.1	48
5. Limestone	3. Limestone	2.6	83	28. Siliceous	1. Limestone	4.8	48
19. Glacial	6. Limestone	3.4	83	28. Siliceous	20. Trap rock	2.8	45
23. Siliceous	7. Limestone	3.7	83	26. Siliceous	17. Limestone	3.4	44
21. Siliceous	20. Trap rock	1.9	82	29. Siliceous	15. Limestone	4.5	44
13. Limestone	16. Limestone	2.2	82	26. Siliceous	6. Limestone	4.1	43
25. Siliceous	20. Trap rock	2.3	82	29. Siliceous	5. Limestone	5.0	43
19. Glacial	17. Limestone	2.7	82	21. Siliceous	18. Limestone	2.9	42
29. Siliceous	22. Quartzite	1.7	81	28. Siliceous	2. Limestone	4.8	42
4. Limestone	2. Limestone	2.6	80	25. Siliceous	14. Limestone	3.6	34
19. Glacial	16. Limestone	2.9	78	24. Siliceous	4. Limestone	4.2	31
29. Siliceous	8. Limestone	4.8	78	25. Siliceous	10. Limestone	3.9	30
24. Siliceous	20. Trap rock	2.2	77	29. Siliceous	14. Limestone	4.4	25
23. Siliceous	16. Limestone	3.4	76	29. Siliceous	4. Limestone	5.2	25
29. Siliceous	9. Limestone	4.8	76	29. Siliceous	10. Limestone	4.7	23
b—Combinations tested in concrete after 21-day curing							
15. Limestone	18. Limestone	2.1	95	29. Siliceous	18. Limestone	0.8	86
7. Limestone	8. Limestone	2.4	93	29. Siliceous	9. Limestone	4.8	86
8. Limestone	20. Trap rock	0.8	93	21. Siliceous	18. Limestone	2.9	80
15. Limestone	20. Trap rock	1.1	93	21. Siliceous	9. Limestone	3.5	80
8. Limestone	9. Limestone	2.4	93	28. Siliceous	19. Syenite	3.0	79
10. Limestone	5. Limestone	2.7	93	29. Siliceous	19. Syenite	3.4	77
10. Limestone	20. Trap rock	0.9	92	20. Syenite	19. Syenite	1.9	72
7. Limestone	20. Trap rock	0.8	91	21. Siliceous	5. Limestone	3.7	70
20. Syenite	20. Trap rock	1.7	87	29. Siliceous	5. Limestone	5.0	68
11. Limestone	13. Limestone	2.2	87				

*For pertinent data on fine aggregates see Table 2.
†For pertinent data on coarse aggregates see Table 3.
‡Δc = difference between coefficients of thermal expansion of mortar and coarse aggregate x 10^6 per deg F.
§Numbers refer to Table 2.
**Numbers refer to Table 3.

effect of this variation is believed to be negligible when extrapolated to the temperatures used in freezing-and-thawing. This follows since only the differences in thermal coefficients are used in the computations, and the difference which is used in the analysis directly as determined remains approximately constant.

TABLE 2—FINE AGGREGATES

Serial No.	Type*	Source	Absorption, percent	Sulfate loss, percent	Coefficient of linear thermal expansion of mortar × 10^6 per deg F.
1. STL-2 G-3 (S)	M Limestone	Prairie du Rocher, Ill.	1.7	5.2	4.4
2. STL-2 G-2 (S)	M Limestone	Krause, Ill.	1.8	5.0	4.4
3. TUL-1 G-17 (S)	M Limestone	Fort Gibson, Okla.	1.1	3.7	4.5
4. TUL-1 G-22 (S)	M Limestone	Fort Gibson, Okla.	1.1	3.7	4.5
5. TUL-1 G-23 (S)	M Limestone	Fort Gibson, Okla.	1.1	3.7	4.5
6. CRD-G-4 (S)	M Limestone	Alton, Ill.	1.1	3.6	4.7
7. CRD-G-6 (S)	M Limestone	Bluestone Dam, W. Va.	0.5	3.2	4.7
8. CRD-G-7 (S)	M Limestone	Wolf Creek Dam, Ky.	0.8	3.7	4.7
9. BUF-4 G-30 (S)	M Limestone	Depew, N. Y.	1.6	3.4	4.7
10. CRD-G-9 (S)	M Limestone	Center Hill Dam, Tenn.	1.0	3.9	4.8
11. TUL-1 G-16 (S)	M Limestone	Fort Gibson, Okla.	1.3	6.0	4.8
12. NED-5 G-1 (S)	M Limestone	Limestone, Me.	2.0	6.9	4.8
13. BUF-4 G-3 (2) (S)	M Limestone	Stafford, N. Y.	1.3	3.0	4.8
14. BUF-4 G-1 (2) (S)	M Limestone	Leroy, N. Y.	1.0	4.5	4.9
15. CRD-G-8 (S)	M Limestone	Dale Hollow Dam, Tenn.	1.3	2.5	5.0
16. STL-2 G-4 (S)	M Limestone	Iron Mountain, Mo.	1.0	2.0	5.1
17. BUF-4 G-2 (2) (S)	M Limestone	Leroy, N. Y.	1.3	3.5	5.4
18. BUF-4 S-9 (2)	N Glacial	Booneville, N. Y.	0.7	5.6	5.5
19. BUF-4 S-2 (2)	N Glacial	Mt. Morris, N. Y.	1.9	10.5	5.5
20. TUL-1 G-24 (S)	M Syenite	Little Rock, Ark.	0.3	1.5	5.6
21. SAC-1 S-1	N Siliceous	Mill Creek, Calif.	1.2	6.9	5.8
22. TUL-3 G-7 (S)	M Limestone	Osage County, Okla.	1.2	—	5.8
23. BUF-4 S-17	N Siliceous	Lancaster, N. Y.	1.4	9.5	6.0
24. KAN-4 S-2	N Siliceous	Harlan County, Kan.	0.7	4.3	6.1
25. STL-2 S-1 + 2	N Siliceous	Mississippi River, Mo., Ill.	0.8	4.7	6.2
26. BUF-4 S-12 (2)	N Siliceous	Lake Erie Sand, N. Y.	1.3	3.8	6.2
27. NED-5 S-1	N Siliceous	Limestone, Me.	2.3	7.0	6.5
28. TUL-1 S-1 (3)	N Siliceous	Van Buren, Ark.	0.6	2.5	6.7
29. CRD S-1	N Siliceous	Hattiesburg, Miss.	0.5	3.2	7.0

*M = manufactured sand; N = natural sand.

TABLE 3—COARSE AGGREGATES

Serial No.	Type	Source	Absorption, percent	Sulfate percent loss	Average coefficient of linear thermal expansion × 10^6 per deg F.
1. TUL-1 G-17	Limestone	Fort Gibson, Okla.	1.1	4.7	1.9
2. TUL-1 G-22	Limestone	Fort Gibson, Okla.	1.1	4.7	1.9
3. TUL-1 G-23	Limestone	Fort Gibson, Okla.	1.1	4.7	1.9
4. KAN-4 G-3	Limestone	Kansas City, Mo.	1.5	20.8	1.9
5. CRD-G-9	Limestone	Center Hill Dam, Tenn.	0.6	2.9	2.1
6. BUF-4 G-30	Limestone	Depew, N. Y.	0.4	1.8	2.1
7. BUF-4 G-1 (2)	Limestone	Leroy, N. Y.	0.5	2.3	2.3
8. CRD-G-6	Limestone	Bluestone Dam, W. Va.	0.5	1.2	2.3
9. CRD-G-7	Limestone	Wolf Creek Dam, Ky.	0.8	12.5	2.3
10. STL-2 G-2	Limestone	Krause, Ill.	1.5	11.0	2.3
11. CRD-G-4	Limestone	Alton, Ill.	1.1	6.3	2.4
12. NED-5 G-1	Limestone	Limestone, Me.	1.2	3.8	2.4
13. TUL-1 G-16	Limestone	Fort Gibson, Okla.	1.0	13.9	2.6
14. STL-2 G-1	Limestone	Falling Spring, Ill.	1.6	10.1	2.6
15. KAN-4 G-2 (2)	Limestone	Amazonia, Mo.	2.6	4.3	2.6
16. BUF-4 G-3 (2)	Limestone	Stafford, N. Y.	0.7	2.3	2.6
17. BUF-4 G-2 (2)	Limestone	Leroy, N. Y.	0.5	5.0	2.8
18. CRD-G-8	Limestone	Dale Hollow Dam, Tenn.	0.8	1.3	2.9
19. TUL-1 G-24	Syenite	Little Rock, Ariz.	0.4	0.6	3.7
20. CRD-G-1	Trap rock	New Haven, Conn.	0.5	0.8	3.9
21. TUL-3 G-7	Limestone	Osage Co., Okla.	1.2	8.0	4.8
22. KAN-4 G-1 (2)	Quartzite	Saline River, Kan.	0.9	6.0	5.4

In addition to computing thermal coefficients of expansion for all coarse aggregates and mortars listed in Tables 2 and 3, the soundness of the individual aggregates and the resistance to freezing-and-thawing of the concretes combinations were determined.

The aggregates were tested in accordance with standard procedures for specific gravity, absorption and soundness by use of magnesium sulfate.

Concrete mixture specifications

The concrete specimens made with these aggregates were prepared from highly standardized and controlled mixtures. The mixtures conformed to the following requirements: (a) a constant water-cement ratio of 5.5 gal. per bag of cement, (b) consistency of 2½- ± ½-in. slump, (c) cement factor chosen to meet consistency requirements and normally falling within the range 5.0–6.0 bags per cu yd, (d) air content of 4.5 ± 0.5 percent, (e) coarse and fine aggregates of specified grading, with 1-in. maximum size and a ratio of sand to total aggregate between 36 and 42 percent by volume, (f) curing for 9 days at 70 F and 100 percent R. H. (except where 21-day curing period was desired for duplicate combinations, and (g) Federal Specifications SS-C-192 Type II cement was used. Using such mixtures, three 3½ x 4½ x 16-in. beams were fabricated for each combination of coarse and fine aggregates. The beams were tested in the automatic freezing-and-thawing apparatus described by Wuerpel and Cook[7] for 300 cycles, or until the average relative dynamic modulus of elasticity reached 50 percent of the initial modulus. The average *DFE* for the three beams of each combination is given in Table 1.

STATISTICAL ANALYSIS AND RESULTS

Preliminary correlation analysis

Evaluation of the effects of various properties on the durability and the determination of the variables most responsible for the differences in *DFE*, was by correlation analysis. Standard statistical methods were used, such as those for multiple correlations found in Ezekiel.[8]

The analysis of the data in Table 1a was based in part on results of preliminary analyses of more limited groups of data which indicated the significant variables for computation, those which could properly be eliminated from the calculations, and the probable advantages of adding other variables. A preliminary analysis of data from 104 combinations of concretes all containing limestone coarse aggregates included the correlation of durability measured as *DFE* with five other variables: air content in percent, cement factor in bags per cu yd, absorption by the coarse aggregates, loss after five cycles of the magnesium sulfate test in percent for the coarse aggregates, and the curing period in days. Of the 104 combinations, 54 contained limestone fine aggregate and the remainder natural siliceous or glacial sands. The basis for developing the relations was a multiple correlation analysis among these variables.

It was easily perceived by inspection, and confirmed by the computations, that the correlations developed were much higher for combinations composed entirely of limestone aggregates than were those for the entire group of

concretes. This fact showed the marked effect of some variable or variables other than those used in the analysis. Since the natural fine aggregates were comparable in quality to the limestone fine aggregates, except for *thermal properties*, it was suggested that the effects of the variation in thermal properties be quantitatively studied. The remaining 54 all-limestone combinations yielded on analysis a relation having a multiple correlation coefficient of $R = 0.891$ which is highly significant.* This correlation showed also that under the rigidly controlled test procedures used, the effects of variation in air content, cement factor, and length of curing were very slight. For these concretes the major factors related to variation in durability were the absorption and loss in the magnesium sulfate test of the coarse aggregates. However, according to K. Mather[9] and others, a high degree of correlation exists between absorption and sulfate loss. This intercorrelation leads to a slightly higher total correlation than would otherwise be the case, but partial correlations show that the effect of absorption is much greater than the effects of sulfate loss. The relations developed for the all-limestone combinations obviously were inapplicable where the fine aggregates consisted of natural sands, but were of use in indicating which variables were nonessential and could be discarded, thus simplifying further computations.

Effect of thermal properties on durability

To demonstrate the effect of thermal properties on durability, a choice of variables best expressing these properties was necessary. In analyzing the combinations in Table 1a, the variables finally chosen were the durability expressed as *DFE*, the difference in thermal coefficient values between coarse aggregates and mortars, Δc, and the absorption in percent, of the coarse aggregates. For ease in computation these variables are assumed to be linearly related to durability. Since this is not strictly true, the results yield a lower correlation than actually exists, but without changing the emphasis of the relations among the variables. The numerical results hold only for the concretes analyzed, but the coefficients of the derived regression equations may be used as a guide for other combinations, with a somewhat greater probability of error. Other variables such as air content and cement factor were not included because of their negligible effect in the range of the mixtures used. Correlation with other thermal properties, such as diffusivity, would be desirable but data regarding these properties were not available for a sufficient number of aggregates. Other differences believed to bear on durability are particle shape and surface texture of the aggregates, but data on these properties are not available in a form suitable for numerical treatment. The analysis was thus restricted in scope.

The 78 combinations used in the final analysis eliminate insofar as possible the effects of extraneous variables such as differences in air content and cement factor. Where duplicate combinations existed the concrete having closest

*In this paper, values for the correlation coefficient which correspond to a probability of chance occurrence of 2 or less in 100 are designated as "significant;" those corresponding to a probability of chance occurrence of 1 or less in 100 are designated as "highly significant."

to the median range of air content was used in the analysis. Those combinations having other than 9-day curing periods were eliminated. The combinations containing limestone coarse aggregates were in most cases represented by more than one freezing-and-thawing test, consequently a single representative test result, having an air content closest to the median value of 4.5 percent, was used. Combinations of other types of coarse aggregate were in most cases represented by only one freezing-and-thawing test.

Final analysis

For the 78 combinations the derived regression equation is:

$$DFE = 109.65 - 8.76\,\Delta c - 15.22 \times \text{absorption (percent)} \quad \ldots\ldots\ldots\ldots\ldots\ldots (1)$$

The coefficient of multiple correlation for this relation is $R = 0.719$. This coefficient is highly significant, as demonstrated by comparison with the $R = 0.20$ coefficient necessary for a 0.95 probability that an actual correlation exists. There is a probability that the coefficient is excessively high but the true correlation of the suite is definitely not less than $R = 0.60$, which is sufficient to show the reality of the general relation among the variables.

To verify that for other sets of concretes the correlation coefficient is of the magnitude found, a rough analysis of 91 concretes was performed. These combinations included 58 of the 78 listed in Table 1a together with an additional 33 duplicates of various of the combinations which varied principally in air contents and length of curing. The coarse aggregates were limestones with manufactured limestone sand and natural sands. The relation derived for these combinations yielded a correlation coefficient of $R = 0.765$, which is not greatly different from the 0.719 determined for the 78 concretes. This similarity in correlation coefficients leads to the placing of a higher degree of reliability on the probable correctness of the results. While exact only for the combinations analyzed, the relations are closely approximate for many other test concretes.

The relations were further analyzed to determine the effects of the coefficient differences and absorptions on the *DFE*'s. The partial coefficients of correlation were computed for the relationship between Δc and *DFE* and between absorption and *DFE* and were determined to be:

$$\Delta c\text{–}DFE \quad 0.5976 \quad \cdot \quad \text{Absorption–}DFE \quad 0.4705$$

Thus the variable of those analyzed most affecting *DFE* is the difference between thermal coefficients of expansion of coarse aggregate and mortar. More than half of the variation in *DFE* accounted for by the relations given for the 78 combinations is directly attributable to this difference. The effect of absorption of the coarse aggregates is also far from negligible. Both of these partial coefficients are significant demonstrating the reality of the effects of these variables on *DFE* and showing that the relations are not due to chance.

Over half of the actual variation in durability may be explained by the variables Δc and absorption alone. This offers promise of assistance in determining the proper aggregate combinations for use in concrete. Thermal

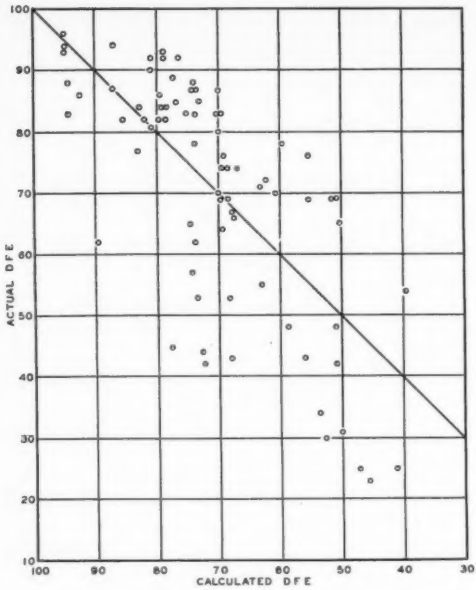

Fig. 2—Comparison of actual and calculated durability factors (DFE) for 78 concretes exposed to accelerated laboratory freezing and thawing

coefficients of expansion and absorption of the coarse aggregates are relatively easy to determine, and can offer valuable indications of the potential suitability of an aggregate combination. However, a *DFE* predicted by substituting the test results in Eq. (1) above is merely a value at which an actual *DFE* either above or below it is equally probable. The standard error of estimate in *DFE* computed from Eq. (1) is 13.55, showing that predictions should not be made on the basis of the absorption and Δc values alone. Fig. 2 is a comparison of actual *DFE*'s with the values derived from Eq. (1). The major value of the relation is its assistance in determining those combinations which require additional testing.

Acceptance of combinations having an acceptable predicted *DFE* is not contemplated without actual freezing-and-thawing tests. While concretes without entrained air were not tested it is believed that the relative importance of the effects of the variables on the *DFE*'s will be retained. The numerical coefficients will change markedly because of the greatly lowered durabilities of concretes not containing entrained air, and the influence of the variables will be enhanced.

A simplified analysis of the 78 combinations was also undertaken, using merely *DFE* and Δc as variables. The relation obtained is $DFE = 100.36 - 10.16\ \Delta c$ (Fig. 3), with a simple correlation coefficient of $r = 0.6165$ which is also highly significant. The linear relation between *DFE* and Δc, assumed for the analysis, is actually quite conservative, since the loss in durability

as measured by the *DFE* increases greater than linearly for increasing Δc. For the 78 concretes, there is no very significant intercorrelation between absorption and Δc, as demonstrated both by the near equality of the partial correlation coefficient and the simple correlation ratio for *DFE*–Δc, and by the actual partial correlation coefficient between absorption and Δc.

PHYSICAL SIGNIFICANCE OF THE RESULTS

Thermal stresses

The statistical analysis demonstrates conclusively the major influence of the coefficients of thermal expansion of the aggregates on the durabilities of various concretes tested in accelerated freezing and thawing. The influence of absorption of the coarse aggregate on the quality of concrete has long been known and needs no detailed discussion. The effect of thermal expansion is little known, and a more detailed study is needed.

Little is known of the stresses created at the boundary of one material embedded in a matrix of another material. With spherical inclusions, the problem has been more extensively studied than for other shapes. Pickett[10] has shown that where a positive volume change takes place in a spherical inclusion, the radial and tangential stresses are approximately

$$S_r = \frac{-2E\delta_a^2}{(1+\mu)\,r^3} \quad \text{and} \quad S_t = \frac{E\,\delta_a^2}{(1+\mu)\,r^3} \qquad (2)$$

Where:

S_r = the radial stress in the matrix, psi.
S_t = the tangential stress in the matrix, psi.
E = the modulus of elasticity, psi.
δ = the radial displacement of the inclusion, in.
a = the initial radius of the inclusion, in.
μ = Poisson's ratio (for normally saturated concrete this ratio may be taken to be 0.25).
r = the radius at which the stresses are computed, taking the origin at the center of the inclusion, in.

The maximum stresses occur at the interface between inclusion and matrix and amount to:

$$S_r\text{max} = -\frac{2E\,\delta}{(1+\mu)a} \quad \text{and} \quad S_t\text{max} = \frac{E\,\delta}{(1+\mu)a} \qquad (3)$$

The radial stress is compressive, and may safely be neglected in the consideration of the thermal effects. The tangential stress, being tensile, is far from negligible as a simple calculation shows.

$$S_t\text{max} = \frac{ET\,\Delta c}{(1+\mu)} \qquad (4)$$

Where:

$\frac{\delta}{a}$ of Eq. (2) and (3) is replaced by $T\Delta c$,

$T\,\Delta c$ = the expansion of the inclusion due to a temperature change of T deg F, where the difference between the coefficients of expansion of the matrix and the inclusion is Δc per deg F, and the other symbols retain the above meanings.

If one assumes Δc equal to 4.0×10^{-6} per deg F, E equal to 5.0×10^{6} psi, μ equal to 0.25, and T equal to 50 F, the maximum tensile stress is 800 psi. This stress exceeds the tensile strength of many concretes and is sufficient to cause cracks. The values used are typical for many limestone-natural sand combinations.

These formulas are derived for spherical inclusions in a matrix of large extent compared to the radius of the inclusion. Equations identical to Eqs. (3) are found for cylindrical inclusions in a similar matrix, with an additional equation $S_z\text{max} = S_t\text{max}$ for axial stress. Since in actual practice these conditions are not met due to particle interaction and the lack of sphericity of the aggregate particles, the above relations are necessarily only approximations. The actual stresses are normally greater than those calculated because of high stress concentrations at the angular edges of many particles. In many cases some lowering of the stresses takes place due to the plastic behavior of the mortar under high stresses. This has not been considered in this discussion, however, since the thermal shock of the freezing-and-thawing tests is sufficient to minimize plastic flow, keeping the mortar in a somewhat brittle condition. Other factors affecting the stresses such as the degree of saturation of the mortar, the effect of bleeding on interfacial bond and the elastic moduli of both mortar and aggregate have not yet been adequately studied.

It is sufficient to state that the stresses created at the mortar-aggregate boundaries are sufficient to cause cracking and distress in concrete when substantial differences between the thermal coefficients of expansion of coarse aggregate and mortar exist. When these tensile stresses are large and cracking occurs, the durability of the concrete suffers due to water entering the cracks and expanding upon freezing, thus further widening the cracks. The concrete thereby loses structural homogeniety, and with repeated cycles of weathering may break into pieces or show other signs of severe deterioration. A loss of bond may also occur when the temperature changes are reversed. When concrete in a structure has decreased from its maximum temperature, following heat evolution due to hydration, to its final temperature, marked tensile stresses between aggregate particles and the mortar may be set up. For example, an aggregate of high thermal coefficient of expansion will cause severe stresses in a less expansive mortar during this change leaving a greatly weakened bond condition, with consequent distress unless relieved by plastic flow.

Maximum thermal effect

The above effects are most marked where the aggregates have quite different thermal expansions, particularly in the case of limestone coarse aggregates and natural sands. The system of limestone, limestone sand and natural sand is of much interest because of the widespread use of these materials in concrete construction. Many relatively pure limestones have thermal coefficients of expansion in the range 2 to 3×10^{-6} per deg F, as determined both by Parsons and Johnson[5] and this study. Limestone-sand mortars normally

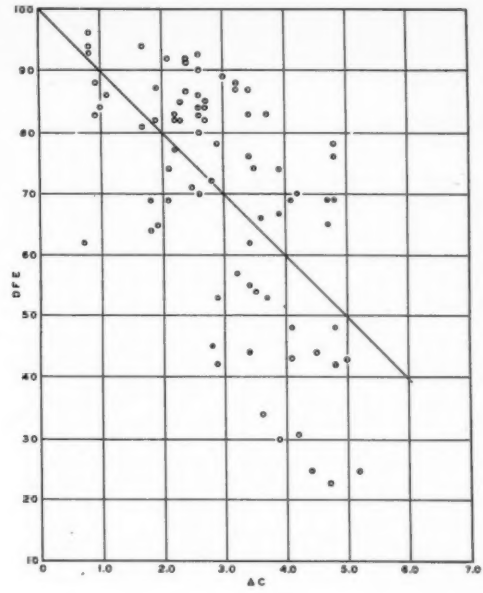

Fig. 3—Relation between durability factors (DFE) of 78 concretes exposed to laboratory freezing and thawing and the differences between coefficients of thermal expansion of coarse aggregates and mortars (Δc)

have coefficients in the range 4 to 5 $\times$ 10^{-6} per deg F while most natural siliceous-sand mortars have coefficients near 6 $\times$ 10^{-6} per deg F and higher. Similar results have been reported by Willis and De Reus.[6] The differences between coefficients of aggregate and mortars are usually about twice as large for the natural sand-limestone combinations as for the limestone-limestone sand combinations, and in conformity with the highly correlated inverse ratio of DFE–Δc, the durabilities of the former combinations in accelerated freezing-and-thawing are much lower than those of the latter (Fig. 3).

Thermal coefficient of mortar

Meyers[12] has shown that the thermal coefficient of portland cement, mortar and concrete, varies with time and moisture content among other things. The tests prescribed above for determining the thermal coefficient of mortar have been standardized to be performed at 7-day age, in a saturated condition. These factors lead to low coefficients for the test mortars as compared with the actual mortars in many concretes not subject to continued curing. Therefore, Δc, determined by the methods of Appendix 1 and 2 is lower and the results are conservative for a variety of normal concretes.

Since the coefficient of expansion of a neat cement paste composed of the cement used in all tests was 6.5 $\times$ 10^{-6} per deg F, it can be seen that for limestone-sand mortars, the lower the cement factor the lower the Δc when these mortars are used in combination with limestone coarse aggregates. It is

advantageous therefore to use a low cement factor in such mixtures. With siliceous sands, the effect of a lowered cement factor is not so great because the difference between coefficients of sand and cement is slight, but a lower cement factor produces less temperature rise, and consequently less expansion, which is desirable to lower the bond stresses. To minimize the differences in coefficients, it is generally desirable to use sands manufactured from the coarse aggregates, particularly when the coarse aggregate has a low coefficient of expansion. The use of natural sands in combination with limestone coarse aggregates should be questioned until tests or service records demonstrate the suitability of the particular concrete mixture proposed.

The effects due to thermally anisotropic aggregates have not been included in this study, since it was found that none of the aggregates possessed such anisotropy to any considerable degree. Use of the average thermal coefficient of expansion was justified for the aggregates selected, but the possibility of distress due to anisotropy should be considered. The possible effects of variation in thermal diffusivities have not been studied, principally due to lack of sufficient data, but they are believed to be much less significant than the effects due to thermal expansion. For a more detailed survey of this problem, see Weiner.[11]

CONCLUSIONS

The series of tests and analyses described indicate the relation of thermal properties of the aggregates to the resistance of concrete to accelerated freezing-and-thawing. Where the difference between coefficients of expansion of coarse aggregate and mortar is large the durability of the concrete may be considerably lower than would be expected from the quality of the aggregates as determined by the usual tests. It is suggested that not only must the individual properties of the aggregates and cement be determined, but the actual concrete mixture must be tested to preclude the use of aggregate combinations with high thermal differential. In general, the accelerated freezing-and-thawing test serves as a valuable indicator of the presence of this phenomenon. Where facilities are lacking for this test, however, the thermal coefficients of both coarse aggregate and mortar may be determined and differences between these coefficients in excess of 3.0×10^{-6} per deg F should lead to caution in the choice of the aggregate combination; further testing, by freezing-and-thawing of concrete, should be performed prior to its acceptance. The tendency exemplified in these tests toward lowered durability is found particularly in limestone coarse aggregate concretes in which natural siliceous fine aggregates are used.

The test for the determination of thermal coefficients for coarse aggregate and mortar is rapid and economical. Even where large differences in the thermal coefficients of the aggregates are not expected, as when sands manufactured from the coarse aggregates are used, it is advisable to perform these tests to prevent possible undesirable properties in the concrete due to the use of thermally anisotropic aggregates.

ACKNOWLEDGMENTS

The preparation of test specimens and the development of the test methods were performed at the Waterways Experiment Station under Herbert K. Cook, Chief, Concrete Research Division. The statistical computations performed by members of the Thermal Research Section are gratefully acknowledged. In particular the writer is indebted to Bryant Mather, Chief, Special Investigations Branch, for valuable assistance in the preparation of this paper.

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APPENDIX 1

DETERMINATION OF COEFFICIENT OF THERMAL EXPANSION OF COARSE AGGREGATE

The samples of rock are sawed on a diamond cut-off wheel into slabs about $1\frac{1}{2} \times 1\frac{1}{2} \times \frac{1}{4}$ in. These slabs are cut parallel to three mutually perpendicular planes from a single piece of rock (Fig. 4). Whenever possible one of these planes is a major structural plane of the rock. By mounting strain gages on these slabs the coefficients are obtained for three mutually perpendicular directions, revealing any anisotropic behavior of the rock. After rinsing in acetone, to remove all traces of the cutting oil, two resistance-wire strain gages are cemented to each slab using Duco No. 5458 cement. The gages are placed parallel to each other along the desired orientation on the face of the slab. A weight of about one pound is placed upon the gages to insure bond and prevent curling, and the prepared slabs are then cured for 24 hours at room temperature. No further polishing of the surfaces after cutting is necessary

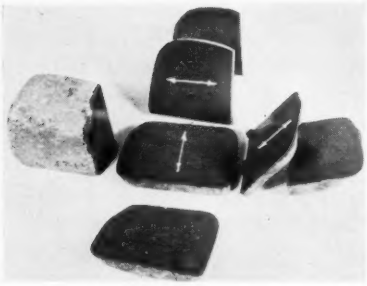

Fig. 4—Orientation directions for strain gages on three mutually perpendicular specimens as sawed from a sample of rock

or desirable for best results. For specimens of the size described, the standard type A-1 gage is suitable. The gages are not reclaimed after use.

After curing, the prepared slabs are mounted in an unrestrained condition on a panel board (Fig. 5). The gage lead wires are attached to binding posts set into the panel and connected through a multiconductor telephone cable to silver-contact, 11-point, 3-bank selector switches. Fig. 6 is a wiring diagram of the installation. The switches are in turn wired to an SR-4 strain indicator. Using two panels with the necessary switches, 72 gages may be read in approximately 10 minutes. Thus 36 slabs or 12 individual rock types may be tested simultaneously. These panels are placed in a controlled-temperature cabinet, operated between 35 and 135 F and at a very low relative humidity. Inside the cabinet, in addition to the specimens, a compensating and an active gage are mounted on a quartz crystal in the direction parallel to the *c*-axis of the crystal and connected to the SR-4 indicator in a manner similar to the specimen gages.

In testing, the temperature of the cabinet is lowered to 35 F and allowed to come to equilibrium as determined by reading of thermocouples located at various points within the cabinet. Strain readings then are taken of all gages by switching individually into the SR-4 indicator circuit. After completing the readings the temperature is raised to 135 F and at equilibrium, readings are again taken. The process is repeated for several cycles. Fig. 7 is a view of the cabinet with control panel and the panel containing the strain indicator, switch box and potentiometer.

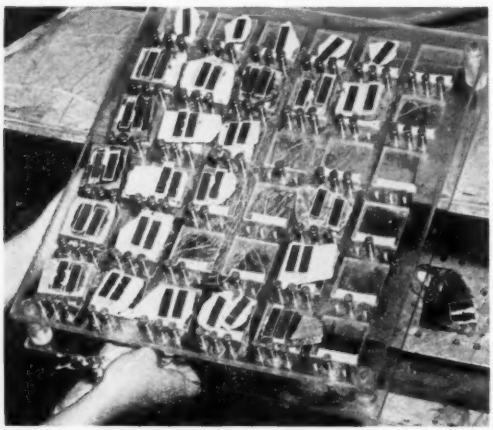

Fig. 5—Arrangement of test specimens with attached strain gages on panel board as placed in controlled-temperature cabinet

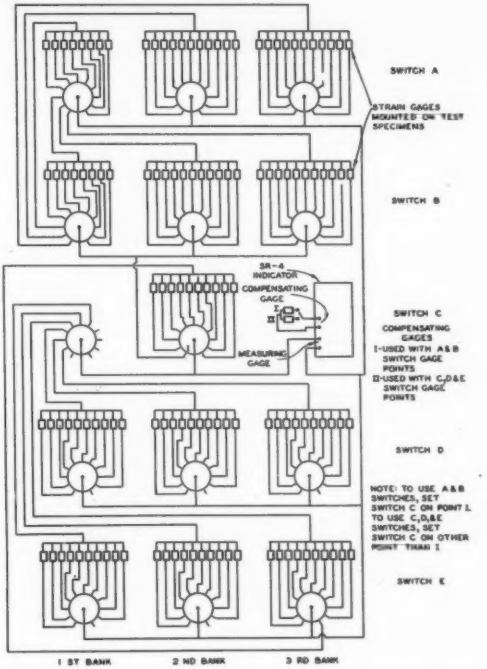
SWITCH A
STRAIN GAGES MOUNTED ON TEST SPECIMENS
SWITCH B
SR-4 INDICATOR
COMPENSATING GAGE
MEASURING GAGE
SWITCH C
COMPENSATING GAGES
1-USED WITH A & B SWITCH GAGE POINTS
2-USED WITH C,D & E SWITCH GAGE POINTS
SWITCH D
NOTE: TO USE A & B SWITCHES, SET SWITCH C ON POINT 1. TO USE C,D,&E SWITCHES, SET SWITCH C ON OTHER POINT THAN 1
SWITCH E
1 ST BANK
2 ND BANK
3 RD BANK

Fig. 6—Wiring diagram for test of coefficient of thermal expansion of coarse aggregate using SR-4 strain gages

The thermal coefficients of expansion for each slab are obtained from the strain readings. Provided there are no influences other than temperature effects, the difference between the readings of the active and compensating gages attached to the quartz crystal should be zero. However, there may be changes in the resistance of the lead wires due to changes in ambient conditions or in other components of the circuit. To compensate for these, the differences actually found between these gages are used as correction factors for the readings of the specimen gages. They are algebraically subtracted from the differences in readings of each specimen gage at 35 F and 135 F. To the corrected readings for the specimen gages, the known thermal coefficient of expansion of the quartz crystal in the direction of its c-axis (4.3×10^{-6} per deg F) is likewise added, thus giving the corrected strain for each slab for the temperature difference between the two readings. The following formula is used for these computations:

$$c = \frac{(4.3\Delta T - \Delta_y + \Delta_x)}{\Delta T}$$

where c = linear coefficient of thermal expansion, $\times 10^{-6}$ per deg F
4.3 = linear coefficient of thermal expansion for quartz along the c-axis, $\times 10^{-6}$ per deg F
ΔT = temperature difference between successive readings, deg F
Δy = difference between successive readings of compensating gage, in. $\times 10^{-6}$ per in.
Δx = difference between successive readings of specimen gage, in. $\times 10^{-6}$ per in.

The linear coefficient of thermal expansion is then equal to the corrected strain divided by the temperature difference. The coefficients so computed are averaged for the two gages on each slab for about ten readings per gage. This average is the linear coefficient of expansion of the rock in the direction in which the gages are mounted. These coefficients are averaged for the three directions giving the over-all coefficient of thermal expansion of the material.

Fig. 7—General view of testing installation showing controlled temperature cabinet and instrument panel

To verify the assumption, inherent in this method, of equal electrical behavior for each switch point and lead wire, and to determine the accuracy obtained, a set of specimens of various materials was prepared and tested according to the procedure given above. Table 4 compares the values for these specimens with handbook values. The accuracy of the method is believed to be within five percent which is quite sufficient for the desired purposes.

This degree of accuracy, together with the large number of specimens which may be tested simultaneously and the rapidity of operation, makes this method well suited for determinations

TABLE 4—COMPARISON OF VALUES OF COEFFICIENT OF THERMAL EXPANSION OBTAINED BY USE OF STRAIN-GAGE METHOD WITH VALUES DETERMINED BY OTHER OBSERVERS*

Material	Coefficient of expansion $\times 10^6$ per deg F	
	Strain-gage method	Other observers†
Copper	9.7	9.3
Aluminum (commercial)	13.0	13.3
Brass	10.3	10.4
Cast iron	5.8	5.9
Glass (soft)	4.4	4.6
Soft iron	6.7	6.7
Lead	15.4	15.1

*These comparisons are only approximate due to possible differences in composition of materials tested.
†*Handbook of Chemistry and Physics*, 25th Edition, 1941, Chemical Rubber Publishing Co., pp. 1644-50.

of coefficient of expansion of aggregates. The complete testing of 12 rock types per week may be accomplished without difficulty. The degree of skill and the equipment required afford wide utility to the method, and for a given time allow a much greater amount of data to be accumulated than do other methods.

APPENDIX 2

DETERMINATION OF COEFFICIENT OF THERMAL EXPANSION OF MORTAR

The specimens used are 1 x 1 x 11½-in. bars, made from a standard mortar mixture, with invar inserts embedded at the ends to give an effective axial gage length of 10 in. A standard mortar mixture was developed on the basis of a series of tests and consists of 564.0 g of cement; 1300.0 g of test sand;* and 274.6 ml of water. To validate the use of such a mixture for all sands, tests were conducted on mortars varying up to 25 percent in sand content, and with from 0.0 to 1.0 ml of air-entraining admixture added per batch. These are much greater variations than those in the concrete mixtures from which specimens are made for freezing-and-thawing tests. The results of these tests on mortar bars are given in Tables 5 and 6. They show that no significant differences are produced in thermal coefficients of expansion of bars made from mortars as a result of variations in sand or admixture content over the ranges indicated above. Therefore, a standard mortar may be used for all tests. The mixture selected was that which gave most satisfactory workability when either natural or manufactured sand was used. The mixing procedure is that of paragraph 8b of ASTM Method C 109, "Standard Method of Test for Compressive Strength of Hydraulic-Cement Mortars." The bars are cast in molds as described in ASTM Method C 151, "Standard Method of Test for Autoclave Expansion of Portland Cement." The bars are hand-tamped into the molds, cured at 70 F and 100 percent relative humidity (R. H.) for 24 hr, stripped and further cured under the same conditions to an age of 7 days.

*Graded as follows:

Sieve	No. 4	No. 8	No. 16	No. 30	No. 50	No. 100
Percent passing	95–100	85–95	75–85	35–55	12–17	3.0–4.5

TABLE 5—COEFFICIENTS OF EXPANSION OF MORTARS* WITH VARYING AMOUNTS OF SAND

Sand, g	Coefficient × 10^6 deg F			
	Sand A†	Sand B†	Sand C‡	Sand D†
1150	5.49	6.17	4.76	6.95
1200	5.58	6.15	4.82	7.02
1250	5.55	6.26	4.87	7.09
1300	5.46	6.14	4.86	7.07
1350	5.49	6.19	4.86	7.03
1400	5.50	6.12	4.81	7.07

*All mortars contained 576.0 g cement, 274.6 ml water
†Natural sands (glacial and siliceous)
‡Manufactured limestone sand

TABLE 6—EFFECT OF AIR-ENTRAINING ADMIXTURES ON COEFFICIENT THERMAL EXPANSION OF MORTARS

Air-entraining admixture, ml	Coefficient × 10^6 per deg F	
	Natural sand mortar	Limestone sand mortar
0	7.0*	4.81
0.05	6.79	4.79
0.10	6.85	4.77
0.20	6.69	4.87
1.00	6.87	4.88

*Made separately

At 7-day's age the mortar bars are placed in a constant-temperature bath maintained at 135 F for 15 minutes, long enough to allow the bars to come to equilibrium. They are then removed individually from the bath, placed in a dial-gage comparator, and a reading taken. This entire operation is completed within 15 seconds after removal from the bath, which is rapid enough so that no appreciable loss of temperature occurs along the major axis of the bar. The bars are then placed in a second constant-temperature bath maintained at 35 F for 15 minutes. The procedure is repeated and readings taken. The difference between readings taken at 135 and 35 F is divided by the product of temperature difference times gage length, to give the linear coefficient of expansion. The coefficients obtained for at least five cycles are averaged, and then averaged for three bars of the same mortar to give the thermal coefficient of expansion of the mortar.

The accuracy of the method is of the same order as the strain-gage method for coarse aggregate, normal inaccuracies in placement of inserts and time lag readings being essentially negligible. At present 24 bars may be tested simultaneously, permitting the determination of thermal coefficients of expansion of eight different mortars per day.

Discussion of a paper by Edwin J. Callan:

Thermal Expansion of Aggregates and Concrete Durability*

By D. L. BLOEM and STANTON WALKER, R. E. GLOVER, T. F. WILLIS and F. V. REAGEL, and AUTHOR

By D. L. BLOEM and STANTON WALKER†

The data presented by Mr. Callan show what appears to be a significant statistical correlation between resistance of concrete to a laboratory freezing and thawing test and differences between thermal coefficients of coarse aggregate and mortar. It is concluded that combinations in which the thermal coefficients of mortar and coarse aggregate differ by more than 3.0×10^{-6} per deg F should be viewed with suspicion. It is believed that the question of whether or not that conclusion is applicable to concrete in actual service should be examined more closely in the light of the test data and conditions of test.

There is always a danger, as Mr. Callan himself has stated elsewhere,[1] that relating of variables by statistical analysis may lead to misinterpretation of their significance if one of the variables happens to reflect the effect of another factor which has not been considered. That appears to be a possibility in this case. Our studies of Mr. Callan's data show a good relationship also to exist between durability factor of concrete and the thermal coefficient of mortar, completely without regard to any characteristics of the coarse aggregate. This is expressed by,

$$DFE = 137.42 - 11.68c_m$$

where c_m is the thermal coefficient of mortar as determined under the selected test conditions. The simple correlation coefficient for this relationship is 0.52 which is not greatly less than the value of 0.6165 shown for the relationship between DFE and Δc to which the author attributes so much significance.

The above analysis strongly suggests that, for these particular data, there was a close relationship between Δc and c_m. The data show that

$$\Delta c = 0.9408\, c_m - 2.4454$$

and that the correlation coefficient for this relationship is 0.70. Table 3 of Mr. Callan's paper reveals why this is so. Of the 22 coarse aggregates selected for study, only two had thermal coefficients greater than 3.9×10^{-6}

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per deg F and these were used in only three of the 78 concretes for which data were analyzed. Since the thermal coefficients of practically all of the coarse aggregates were quite low and covered a narrow range (only 2.0×10^{-6} per deg F for 75 of the 78 concretes), it follows that variations in Δc were a reflection almost entirely of the changes in thermal coefficients of mortar.

Unfortunately, the data analyzed by Mr. Callan do not include thermal coefficient measurements of the concrete itself. However, it has been shown[2,3] that the thermal coefficient of concrete is a direct function of the thermal coefficients of its ingredients. Therefore, because of the limited range in thermal coefficients of coarse aggregate in this case, it would be expected that the thermal coefficients of concrete would vary regularly with those of the mortars used. The data, then, show rather conclusively that resistance to rapid freezing and thawing in these tests was related to at least three variables connected with thermal properties: (1) difference in thermal coefficient between coarse aggregate and mortar, as demonstrated in the paper; (2) thermal coefficient of the mortar alone; and (3) thermal coefficient of the concrete. Since, as has been shown, these variables were closely interrelated in this case, the problem reduces to selecting the one which most significantly influenced resistance to rapid temperature changes employed in the tests.

The stress analyses shown by Mr. Callan appear to offer convincing support for No. 1. The formula used indicates that, for a Δc of 4.0×10^{-6} per deg F and a temperature change of 50 F, the theoretical tensile stress would be 800 psi. Note, however, that under these conditions the differential strain developed between the mortar and coarse aggregate is only 0.0002 in. per in. It appears to follow, according to the formula, that any phenomenon which would cause a differential strain of 0.0002 or more would result in excessive stresses in the mortar. What then of other volume changes such as those resulting from variations in moisture content which are characteristic of all concretes? It has been shown that a change in moisture of one percentage point may cause a length change of as much as 0.0002 in. per in. and that, therefore, moderate cyclic moisture changes can easily account for length changes of 0.0006 in. per in. or more in the mortar. Since the coarse aggregate volume would be affected little by change in moisture, the resultant tensile stress (for the E of 5×10^{-6} psi assumed in the calculation) would be 2400 psi or three times as large as that resulting from the rather large temperature change and difference in thermal coefficient used in the author's calculation. Since concrete withstands moisture changes of considerable magnitude and frequency, usually without distress, it appears that some sort of plastic deformation must serve to relieve the stresses theoretically developed. There seems no reason to suppose that much smaller stresses resulting from differential thermal volume changes would not be similarly relieved.

The formula for theoretical tensile stress resulting from differential thermal expansions

$$S_t \text{ max.} = \frac{ET\ \Delta c}{(1 + \mu)}$$

presents some further difficulties. It shows stress to be independent of particle size. Why, then, be more concerned about differences between thermal coefficients of coarse aggregate and mortar than between fine aggregate and cement paste? In both cases the failure must occur ultimately in the cement matrix and adjustments due to plastic flow would be expected to be about equal. Since the thermal coefficient of hardened cement paste almost always exceeds 6×10^{-6} per deg F, it would appear to be impossible to produce a mortar with a low coefficient fine aggregate that would not develop excessive stresses upon temperature change. Such combinations are, of course, successfully used, as are combinations of mortar and coarse aggregate differing widely in thermal coefficient. The evidence, therefore, indicates that concrete can adapt itself to differences in volume change characteristics of its ingredients. Differences in thermal coefficient do not appear to have been, in themselves, a major factor in resistance to temperature changes in these tests.

What, then, of the other two possible factors—thermal coefficients of mortar and concrete? For the data in question, both of these can be considered essentially the same since they were so closely related. Tests reported by Walker, Bloem and Mullen,[2,3] which were made for the specific purpose of evaluating effects of thermal characteristics, showed that resistance of concrete to rapid temperature changes was progressively reduced as the thermal coefficient of concrete was increased. No relationship was found between resistance to temperature changes and differences between thermal coefficients of coarse aggregate and mortar. It was concluded that rapid temperature changes resulted in steep temperature gradients within the specimens with resultant differential length changes at different depths. The magnitude of induced stresses would depend upon the thermal coefficient of concrete and account for the differences in resistance.

In the tests reported by Mr. Callan, a similar situation appears to have existed. Although thermal coefficients of the concrete were not measured, the good relationship between *DFE* and thermal coefficient of mortar suggests that resistance to rapid temperature changes was a function of the thermal coefficient of concrete. The relationship found between *DFE* and Δc can be assumed to be secondary and to have resulted from the close correlation between Δc and thermal coefficient of concrete for the range of variables selected for study.

Finally, it must be kept in mind that the conditions of test were such as to accentuate the effects of concrete thermal coefficient on measured resistance to temperature changes. Temperature changes were very rapid, resulting in steep temperature gradients within the specimens and large differential strains—a factor of particular importance when tensile stresses are induced in the surface, as in the rapid freezing cycle. In most field exposures it is unlikely that temperature differentials would be of sufficient magnitude to cause these excessive strains in the concrete.

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3. Walker, Stanton, Bloem, D. L., and Mullen, W. G., "Effects of Temperature Changes on Concrete as Influenced by Aggregates," ACI JOURNAL, Apr. 1952, *Proc.* V. 48, p. 661.

By R. E. GLOVER*

In his paragraph on "Thermal Stresses" the author presents formulas for computation of radial and tangential stresses adjacent to the boundary of a spherical inclusion (Eq. 3). He then states that the radial stress is compressive and may safely be neglected but that the tangential stress, being tensile, is not negligible. The writer would object to these statements because the coefficient of thermal expansion of aggregate could be either less or more than that of the matrix and because the temperature change could be either an increase or a decrease. In two of the four combinations which these possibilities would provide, the author's statement would be true but in the remaining two, the conditions would be reversed. Radial tensile stresses and compressive tangential stresses would then be present.

The author's Eq. 4 appears to be obtained from the second of Eqs. 3 by introducing the assumption that the interference produced by a temperature change acting on the difference of the thermal expansions of the inclusion and the matrix occurs without modification due to the mutual restraints which the inclusion and the matrix exert upon each other. By including these mutual restraints, the writer has obtained the relations

$$S_r = \frac{T(C_2 - C_1)}{\frac{(1 + \mu_2)}{2E_2} + \frac{(1 - 2\mu_1)}{E_1}} \quad \ldots\ldots (1)$$

$$S_t = -\frac{T(C_2 - C_1)}{\frac{(1 + \mu_2)}{E_2} + \frac{2(1 - 2\mu_1)}{E_1}} \quad \ldots\ldots (2)$$

The author's notation has been followed as closely as possible in presenting these expressions but with the understanding that S_r and S_t are positive if tensile. The subscripts 1 and 2 refer to the inclusion and to the matrix respectively. The quantity C represents the thermal coefficient of expansion, which is positive if an increase of temperature produces expansion. The temperature change T is positive if it represents a temperature rise.

For the special case discussed by the author where

$$\mu_1 = \mu_2 = \mu \quad \ldots\ldots (3)$$

$$E_1 = E_2 = E \quad \ldots\ldots (4)$$

The expressions became

$$S_r = \tfrac{2}{3} \frac{ET(C_2 - C_1)}{(1 - \mu)} \quad \ldots\ldots (5)$$

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$$S_t = -\tfrac{1}{2}\frac{ET(C_2 - C_1)}{(1 - \mu)} \quad \ldots\ldots (6)$$

With $\mu = 0.2$, the tangential stress given by this last expression would be numerically one half that given by the author's Eq. 4. The radial stress would be numerically twice the tangential stress but of opposite sign.

The writer concludes that radial tensions are possible, that they have an intensity which is approximately twice that of the corresponding tangential stresses, and that their effectiveness in producing disruption of concretes subjected to temperature changes could exceed that of the tangential stress components.

By T. F. WILLIS and F. V. REAGEL*

Mr. Callan's subject is one that has for some time intrigued a number of concrete technologists. The general tenor of the ideas offered in the past suggested that "thermal incompatibility" of the constituents of concrete, *i.e.*, a difference between the thermal coefficients of expansion of (a) the paste and fine aggregate, (b) the paste and coarse aggregate, or (c) the mortar and coarse aggregate, might be a major factor in determining concrete durability. Prior to publication of Mr. Callan's paper the idea was presented as an hypothesis, and the small amount of accompanying experimental evidence was offered as only an indication, not proof, of its validity.

Although the present writers are not certain of the author's intent, we believe the paper might be construed by many readers as purporting to prove one facet of the hypothesis; namely that *the difference between thermal coefficients of expansion of mortar and coarse aggregate (designated Δc in the paper) has a decided effect on the durability of concrete in service, regardless of the mineralogical types of the aggregates.*

If the effect of Δc on the durability of concrete has been removed from the hypothetical category and placed in that of proven facts by Mr. Callan's paper, there has occurred an event of considerable significance†—not only to the concrete industry but to the aggregate industry as well. It could cause a change in the thinking of concrete technologists and work radical changes in the economics of the aggregate industry, imposing the threat of extinction on a considerable segment of the latter. On this score alone the paper warrants careful study. Furthermore, an element of doubt as to the existence of a true correlation between durability of concrete and Δc was injected into the writers' minds by actual service records of concrete structures, as contrasted with laboratory tests. When we considered our studies of 20 to 30 year old Missouri pavements, together with our knowledge of the thermal coefficients of various aggregates used and the indicated values of Δc, we were immediately faced with two facts which tended to negate the Δc hypothesis:

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†Throughout this discussion when the words "significant, significantly, and significance" are uncapitalized, they carry only their ordinary meaning; if capitalized they carry the additional connotation "as determined by standard statistical tests."

(1) We have about 100 miles of pavement in one section of the state, all of which contains the same fine aggregate, a highly siliceous material colloquially designated as "flint." This is a manufactured fine aggregate obtained by crushing larger pieces. About one-half of this pavement mileage contains 2-in. crushed limestone as coarse aggregate and the other half contains 2-in. crushed flint, the parent material from which the sand for the entire mileage was manufactured. Δc for the flint sand-limestone concrete would be about 3.5; for the flint sand-crushed flint concrete it would be about 0.2 to 0.5. The difference in deterioration ascribed to weathering, both in rate and quantity, has been marked. But the difference does not conform to Mr. Callan's hypothesis; in fact it is in exactly the opposite direction. The performance of the flint sand-limestone concrete has been outstandingly good; whereas that of the flint sand-crushed flint concrete has been generally poor.

(2) In this state there are about 3000 miles of pavement 20 or more years old all of which contain natural sands composed primarily of siliceous material. About 60 percent of this mileage contains various limestone coarse aggregates having thermal coefficients ranging from 2.5 to 4.3×10^{-6}. These combinations would have Δc's of 2.5 to 4.0. Without exception, where sections of such pavement are comparable as to service conditions with other sections whose aggregate combinations would result in lower Δc's, the former have shown materially less deterioration due to weathering.

For these reasons the writers have considered the data of the paper, together with the conclusions and implications, with more than ordinary interest.

Because the author's idea of the scope of applicability of his conclusions is not clear to the writers, this discussion is divided into three phases. Each successive phase covers a discussion of the justification for applying certain of the author's conclusions to successively broader ranges of the problem of selection of aggregates for durable concrete and of predicting durability from laboratory freezing and thawing tests.

PHASE I

Assuming that the author intended his conclusions to apply *only to the specific concretes and aggregate combinations used in the experiment, and to the deterioration measured by DFE and caused by the extremely accelerated freezing method used,* is he justified in concluding that the *DFE* is significantly affected by Δc?

Mr. Callan's conclusion is based on results of his statistical analysis. The fundamental equations used for calculating regression equations, correlation coefficients, and tests of the Significance of both, were originally derived under certain well defined limitations pertaining to the universe of data and the manner of selecting variables. If, in an actual experiment, the data do not conform approximately to the limitations assumed for the basic equations, a calculated correlation coefficient may be without practical significance —even though its numerical magnitude may indicate it to be highly Significant. There are several points at which Mr. Callan's data diverge widely from some of the assumed limitations. To explain these divergences and their effects would require an involved exposition of theoretical statistics, a task which the writers would approach with some reluctance.

A few calculations, using the author's data, and a little deductive reasoning should indicate the degree of significance that can be attached to the multi-

ple correlation coefficient, $R_{DFE,\ \Delta c,\ percent\ absorption}$, on the high value of which the author bases his conclusion. With the hope of making the implication of this phase clearer to the reader, we shall adopt the artifice of supposing that we had had access to Mr. Callan's experimental data but not to his interpretations. After due deliberation we select c_m (thermal coefficient of mortar) and percent of absorption of the coarse aggregates as the measured variates most apt to determine the magnitude of *DFE*. Next we calculate a regression equation and multiple correlation coefficient with the results shown in line 2 of Table A. Using Mr. Callan's criterion of less than 1 in 100 chances that a random sample of 78 items from an uncorrelated universe would show a value as high as was obtained, we may regard the correlation as highly Significant.

However, any correlation coefficient may have a magnitude which indicates it to be significant in a statistical sense, but the correlation may be illusory rather than real. Therefore, consideration must be given to other possibilities before we may categorically accept the evidence for this tentative conclusion.

Study of the author's sampling design shows us that of the 78 concretes tested, the 25 concretes with mortar thermal coefficients in the lower third (4.4 to 5.2 $\times$ 10^{-6} inclusive) of the entire range all contained limestone fine aggregate; and the 28 concretes with mortar coefficients in the upper third (6.2 to 7.0 $\times$ 10^{-6} inclusive) all contained siliceous fine aggregate. Of the remaining 25 concretes, with mortar thermal coefficients in the middle third of the range, some contained limestone, some siliceous, and some miscellaneous fine aggregates. For a calculated regression equation and correlation coefficient, the points near the extremes of the range of the independent variable will usually have a much greater effect on determining the constants than do the points near the center of the range. That the central points of the author's data do not materially affect the regression line was checked by eliminating the concretes having mortar thermal coefficients of 5.3 to 6.1 $\times$ 10^{-6} inclusive and calculating a regression equation (relating *DFE* to thermal coefficient of mortar and absorption percentage of coarse aggregate) with the result shown in line 4 of Table A. It is obvious that the constants of this equation and the correlation coefficient are not Significantly different from those of the equation which was based on all the data.

Thus the regression equation (*DFE* $-$ c_m $-$ percent absorption) and the multiple correlation coefficient based on all the data is probably determined

TABLE A

Line No.	Variates involved	Regression equation	Correlation coefficient
1	*DFE* $-$ Δc $-$ % Abs.*	*DFE* = 109.65 $-$ 8.76 Δc $-$ 15.22% Abs.	0.719
2	*DFE* $-$ c_m $-$ % Abs.*	*DFE* = 148.214 $-$ 10.70 c_m $-$ 18.68% Abs.	0.697
3	*DFE* $-$ Δc $-$ % Abs.†	*DFE'* = 112.305 $-$ 9.17 Δc $-$ 15.88% Abs.	0.756
4	*DFE* $-$ c_m $-$ % Abs.†	*DFE* = 151.92 $-$ 10.805 c_m $-$ 21.19% Abs.	0.749
5	Δc $-$ c_m*	Δc = $-$ 2.446 + 0.941 c_m	0.695

*Calculated from all the data in Table 1a of the original paper.
†Calculated from the data of Table 1a, excluding all concretes containing mortars with thermal coefficients of 5.3 to 6.1 $\times$ 10^{-6} inclusive.

largely by the fact that the author's sampling design used only limestone fine aggregate for obtaining mortar thermal coefficients near one extreme, and only siliceous fine aggregate for values near the other extreme of the entire experimental range of mortar thermal coefficients. Graphical studies of the sample design verify this.* If the thermal coefficient of mortar was the only attribute of concrete, which would be pointedly affected by the difference between inherent characteristics of limestone and silica, and which might logically be expected to influence the *DFE* of concrete, we need not be perturbed by such a sampling design. There are, however, at least three other attributes which would vary qualitatively, if not quantitatively, in the same manner exhibited by the thermal coefficient of mortar, *viz.*, (a) the thermal coefficient of concrete, (b) the elastic modulus of mortar, and (c) the elastic modulus of concrete.

Furthermore, in the writers' opinion, there is fully as much *a priori* justification for assuming that any one of the above, rather than the thermal coefficient of mortar, is the truly correlated factor. Therefore, with only the data of the paper available, our own tentative conclusion, that the *DFE* is truly correlated with the thermal coefficient of mortar, cannot be verified. (Through analagous studies and reasoning, identically the same conclusion can be reached for Δc.)

At this point suppose we learn of Mr. Callan's idea that *DFE* is dependent on the magnitude of Δc and decide to investigate this possibility. Preliminary to calculating the correlation coefficient (*DFE* $-$ Δc $-$ percent absorption) we check the correlation between Δc and c_m with the results shown in line 5 of Table A. This correlation is of course highly Significant, as would be expected from the algebraic relation of Δc and c_m. Since for the data of the paper *DFE* and Δc are each highly correlated with c_m, it therefore seems logical to predict that *DFE* will be highly correlated with Δc. However, we verify this prediction by determining the correlation coefficient *DFE* $-$ Δc $-$ percent absorption, which is shown in line 1 of Table A, and find it to be highly Significant.

Now, we find ourselves in a quandary. We have obtained correlations, *DFE* with c_m and with Δc, which by statistical test are both highly Significant. Are they both real or is one or both illusory? Considering the second preceding paragraph, it appears that the sample design used in collecting the data of the paper is inadequate for resolving this problem. We are then faced with the fact that, with only the data of the paper available, there is no more justification for concluding the *DFE* is truly correlated with Δc than there is for concluding that the true correlation exists between *DFE* and c_m.

This anomalous condition exists because of an imcomplete sample design. Δc is a magnitude, any specific value of which can be obtained by a variety of combinations of sand and coarse aggregates. When an experiment for determining the correlation of two attributes of a material involves use of

*It may also be demonstrated through application of either Pearson's or Yule's test for association of species in an experimental sample.

combinations of two or more species for obtaining different values of one of the attributes, it is imperative that the experimental sample be representative of all possible combinations of the species. The necessity may be illustrated by considering a universe of concretes containing all combinations of only two mineralogical species of aggregates, limestone and siliceous. The combinations of species listed in column 2 of Table B are possible.

In Table B column 3 gives the qualitative relationship between the Δc's that would be expected from the combinations shown in column 2. Column 4 shows the relative qualitative magnitude of *DFE* that would be expected under the hypothesis that Δc is a major determinant of *DFE*. Column 5 shows the qualitative magnitudes of *DFE* actually obtained by Mr. Callan. Column 6 shows the qualitative relationship between the c_m's for the combinations shown in column 2. Column 7 shows the qualitative magnitudes of *DFE* that would be expected under the hypothesis that c_m (rather than Δc) is a major determinant of *DFE*.

Now if we compare column 4 with column 7, we note that for two aggregate combinations, (a) and (c), columns 4 and 7 are identical. In other words, with these two aggregate combinations the relative *DFE*'s would be the same; regardless of which hypothesis is correct. We further note that these are the aggregate combinations that the author used, and that it is therefore impossible to determine from the author's data which of the two hypotheses is correct, or even if either is wrong.

Comparison of columns 4 and 7 for aggregate combinations (b) and (d) reveals that if the Δc hypothesis is true the relative *DFE*'s of combination (b) and combination (c) would be exactly opposite to that which would be expected if the c_m hypothesis were the correct one. It therefore appears that if combinations (b) and (d) had been included in the experiment, one of the hypotheses could have been proved wrong; but even this would not have proved the other to be right.

All of the discussion in "Phase I" is, in a sense, negative.* It does not prove that *DFE* is independent of Δc; it does show that even limiting the conclusion to the author's aggregate combinations when tested under his specific laboratory procedure the high value of the calculated correlation coefficient does not necessarily indicate that *DFE* is in any degree dependent

TABLE B

(1) Comb. No.	(2) Aggregate		(3) Value Δc	(4) Expected value of *DFE*	(5) Actual value of *DFE*	(6) Value of c_m	(7) Expected value of *DFE*
	Fine	Coarse					
(a)	Limestone	Limestone	Low	High	High	Low	High
(b)	Limestone	Siliceous	High	Low	—	Low	High
(c)	Siliceous	Limestone	High	Low	Low	High	Low
(d)	Siliceous	Siliceous	Very low	High	—	High	Low

*As a matter of fact, the use of statistics for testing any hypothesis results in a similar impasse. If the statistical tests of a proper universe of data so indicate, one can state that a hypothesis has been disproved and be relatively certain of being correct. However, because of the ever-present possibility of choosing attributes which give an illusory correlation, the odds are stacked against a statement—based only on statistical tests—that a hypothesis has been proved.

Such proof can be established by the fact that replicate test results on different populations of data are found to be in agreement with predictions *based on the hypothesis and made prior* to the tests.

on the magnitude of Δc. In other words, it has not been proved that the difference in the thermal coefficients of the mortar and coarse aggregate was, *per se*, a major determinant of the *DFE* of the concrete.

PHASE II

Assuming that the author intended his conclusions to apply to that universe of concretes made up of all possible combinations of the mineralogical types of aggregates used in his experiments, is he justified in concluding that *DFE* is substantially dependent on Δc?

It is obvious that any conclusions, which are not justified for a universe confined to the author's experimental sample (which we attempted to show in "Phase I"), should not be applied to a more extensive universe. Even in the case where conclusions are demonstrably applicable to the data of an experimental sample, it is always dangerous to extrapolate such conclusions into realms not represented in the sample. In the writers' opinion, had the author included in his experimental sample a number of aggregate combinations such as siliceous coarse aggregate-siliceous fine aggregate, limestone fine aggregate-siliceous coarse aggregate, he might have obtained a correlation coefficient (DFE–Δc) which would have indicated little or no significant correlation.

PHASE III

It is the ultimate purpose of any experiment to establish a basis for action. The obvious action to base on experimental results such as the author's is prediction of the durability of a concrete in service from knowledge of the thermal expansivities of the aggregates and of the *DFE* determined by the accelerated laboratory freezing and thawing test he used. No doubt, the author did not intend that his conclusions be applied so broadly; nevertheless, after reading the paper, several concrete technologists (other than the writers) thought the author gave the definite impression that the magnitude of Δc would generally be an important factor in controlling durability of concrete in service. If this impression be widespread the following discussion should be pertinent.

The point made in "Phase II" militates against use of the author's experimental results for supporting the thesis that a DFE–Δc relation holds for a further enlarged universe of concretes. But even if a subsequent series of tests should prove that there is such a relation, another element of information would have to be obtained before it could be established that Δc is a controlling factor in determining durability of concrete in service. The missing element is a determination of what, if any, correlation exists between *DFE*—as measured under the author's laboratory freezing and thawing procedure—and deterioration of concrete in service.

Several investigators, among them the writers, have pointed out the possible divergence in results obtained in the laboratory using a relatively rapid rate of cooling and those resulting from the rather slow rate characteristic of that occurring under most service conditions. So far as the writers are aware,

no one has shown that such a correlation exists. Certainly there are numerous bits of experimental evidence, as well as a definite theoretical basis, for doubting its existence. Therefore the present seems premature for placing too much credence in such laboratory test results for predicting either the durability of a concrete under service conditions or the effect of the attributes of the concrete components on such service durability.

In closing the writers wish to emphatically disclaim any idea that this discussion disproves the hypothesis regarding the possible effects of thermal expansivities of concrete components on durability of concrete. We do believe it to show that any current statement about such effects still belongs in the realm of the hypothetical; and that, for the routine selection of aggregates for job concrete, there is insufficient factual information presently available to warrant discrimination against any aggregate combination solely on the grounds that it would have a high value of Δc.

AUTHOR'S CLOSURE

The author is gratified to learn both by these discussions and from other sources of the widespread interest in this problem of relating thermal properties of concrete to durability. It appears that some clarification of the intent and implications of the paper would be desirable. The paper was intended to present results of tests conducted, together with simple methods for determining the pertinent thermal properties; to discuss these results in terms of stress attendant upon differential thermal expansion; and to serve as an incentive for additional work along these lines, so as to delineate the effects of this phenomenon in its relation to other deteriorating influences. It was not intended to state with finality that concretes having such large differential expansions should not be used, nor to imply that the effects of this lowered durability were necessarily overbearing in comparison with other effects. That such implications may have been drawn is indeed unfortunate, and the author is glad for the chance to correct them.

In considering effects of thermal properties on concrete durability, apart from other factors such as absorption, it seems reasonable to assume that reduction in durability is related to stresses produced in concrete by virtue of temperature changes. Such stresses could be caused by the action of several of the thermal properties.

(a) Stresses due to uneven heating or cooling of concrete as a whole, depending on the coefficient of expansion of concrete.

(b) Stresses due to differential expansion of the components of concrete during a temperature change.

(c) Stresses due to uneven heating or cooling of the components of concrete, by virtue of differential thermal diffusivities, during a temperature change.

Stresses due to uneven heating, we may compute using the methods given by Timoshenko.[1] For the case of a slab initially at constant temperature T_0, suddenly surface cooled to temperature T_1, we have the stresses

$$\sigma = \frac{4\alpha E (T_0 - T_1)}{\pi (1 - \mu)} \sum_{n=1}^{\infty} \left[\frac{2}{(2n-1)\pi} - \cos \frac{(2n-1)\pi y}{2d} \right] e^{-(2n-1)^2 \pi^2 \alpha t / 4d^2}$$

Where σ = stress in direction normal to y
α = coefficient of expansion
E = modulus of elasticity
μ = Poisson's ratio
T_0 = initial temperature
T_1 = surface temperature ($t > 0$)
y = distance from midplane of slab
d = distance from midplane to surface
κ = thermal diffusivity
t = time

Inserting values of $\alpha = 5 \times 10^{-6}$ per deg F, and 4×10^{-6} per deg F, respectively, $E = 5 \times 10^6$ psi, $\mu = 1/4$, $T_0 - T_1 = 40$ F, $d = 2$ in. (4 in. slab), and $\kappa = 0.05$ sq ft per hr, we obtain the following approximate stresses 5 min. after the surface temperature changes have occurred for the two concretes of different coefficients of expansion.

Location	Tensile stress, psi	
	$\alpha = 5 \times 10^{-6}$ per deg F	4×10^{-6} per deg F
At surface	750	600
½ in. below surface	300	240
0.88 in. below surface	0	0

These stresses are relatively high near the surface but decrease quite sharply and are compressive in the center of the slab. Average stress over the thickness of the slab is $\int_o^d \sigma \, dy = 0$. For lower coefficients of expansion and modulus of elasticity stresses would likewise be lowered proportionally. Since the differences in over-all concrete coefficient due to different mortars would be of the order of 1×10^6, based on Fig. 4 of Walker, Bloem and Mullen's paper[2] for 40 percent coarse aggregate by volume, the variation in stress between concretes made with different mortars would thus be only of the order of 60 psi ½ in. below the surface. This treatment does not however include the insulating effects of surface films which are present in the test procedures and would lower the stresses considerably. Such stresses could play some role in the deterioration as suggested by Messrs. Bloem and Walker. These differences in stress due to coefficient of expansion differences could affect the surfaces, but are hardly significant in the interior.

Next considering the stresses due to differential expansion of the concrete components, they are described simply in the paragraph on thermal stresses (p. 495). As Mr. Glover has pointed out in his discussion, the effects of aggregate compressibility were not included in these equations. When these effects are taken into consideration, considerable changes in the stresses occur, as shown by Mr. Glover and also Pickett[3] and Robinson.[4] For the case of the spherical inclusion, Robinson has shown that the sum of the principal stresses in the interior is, using our notation:

$$\Theta = S_r + 2S_t = \frac{6 G T \Delta c}{1/2 + \frac{G (1 - 2\mu')}{G' (1 + \mu')}}$$

where G and G' are the shear moduli of mortar and aggregate respectively and μ' is Poisson's ratio of the aggregate. This reduces to

$$\Theta = \frac{6\,E\,E'\,T\,\Delta\,c}{E'\,(1+\mu) + 2E\,(1-2\mu')}$$

with E and E' the Young's moduli of mortar and aggregate. It can be seen that as the aggregates and mortars used are more rigid (G' and G, and thus E' and E higher) the stresses become higher for the same temperature changes. These effects will have quite a bearing on the concrete stresses.

For example, with an aggregate of Young's modulus = 7.5×10^6 psi, a change in the Young's modulus of the mortar from 1×10^6 to 2×10^6 psi would result in an increase of 80 percent in stress. It should be noted also that the equations of the paper were specified for the case of a positive volume change in the inclusion, and led to lower stresses than the higher radial stresses for negative volume changes, as shown by Mr. Glover. From Pickett[3] and Mr. Glover's Eq. 6, for $E_1 = E_2 = E$, and $\mu_1 = \mu_2 = \mu$ (= 0.25) the tangential tensile stresses would be 4/9 of those computed in the paper.

It is also worthy of note that Mindlin and Cheng[5] have shown that for the case of a spherical inclusion near an edge the stress may reach a value of $4(1 + \mu)$ times the stress on a similar inclusion embedded in an infinite matrix. Thus, for the constants chosen in the paper, and including compressibility effects, the stress on the aggregate near the edge would be $4(1 + 0.25) \times 4/9 \times 800 = 1775$ psi. However, the effects due to shape variations in aggregate particles with high stress concentrations and the effects due to the large number of embedded particles would cause complexities which it was desired to avoid in the paper. The stresses were presented merely as an indication; and the many other effects present would reduce the value of performing additional mathematical operations without better experimental data.

Stresses due to uneven heating or cooling of concrete components, by virtue of differential thermal diffusivities, has been discussed by Weiner[6] and has not been extensively considered for the paper.

Messrs. Bloem and Walker have stated: "It has been shown that a change in moisture of one percentage point may cause a length change of as much as 0.0002 in. per in., and that, therefore, moderate cyclic moisture changes can easily account for length changes of 0.0006 in. per in. or more in the mortar." It has been determined* that the change in moisture referred to is in total moisture content of the mortar, that is, weight of water per total weight of mortar, and also that the cyclic changes referred to are seasonal types for an exposed structure rather than rapid changes in laboratory tests.

Because of the marked effect on shrinkage due to coarse aggregate,[7] it is impractical to separate the length changes in the mortar from the concrete as a whole. Since the difference in total moisture content of concrete in passing from a dry to a saturated state is only a few percent, say, from 1 to 6 percent, the changes described would be rather severe. In addition such changes would occur over a considerable period of time, due to the relatively slow

*Private communication with Stanton Walker.

diffusion of moisture in concrete as compared with rapid heat diffusion, permitting greater dissipation of the induced stresses by plastic flow. The stresses are, however, in many instances[7] sufficiently high to cause cracking of the concrete, both surface cracking and internal cracking. In determining these stresses, equations similar to those of (a) and (b) would be involved, but the time factors would be dissimilar due to the slower moisture diffusion. It was noted in the paper that the stresses may be lowered due to plastic flow, but this stress relief is minimized in the freezing and thawing tests described, while it may be prominent in the case of the cyclic moisture changes. As to the effects of stresses with respect to particle size, Pickett[3] has shown the stresses to be higher near smaller particles, but with a much lower effective radius, which would lead to considerably more plastic flow in the mortar as a whole, than in the mortar-coarse aggregate system.

It is indeed possible that a portion of the decrease in durability was attributable to the thermal coefficient of the concrete as a whole, as suggested by Messrs. Bloem and Walker, and also by Messrs. Willis and Reagel. It is not felt, however, that the contention of Messrs. Bloem and Walker that the DFE–Δc relationship is secondary and fortuitious is justified. For the data of the paper, it is apparent from Table 1 that in 15 of the 17 cases where a coarse aggregate was tested both with sand manufactured from it and with other sands, the combination of aggregate and sand manufactured from it had higher DFE and lower Δc. The other two cases showed a higher DFE with a higher Δc, but the difference in DFE for these concretes with different sands amounted to only 2 points in 86 and 87 respectively, which difference is not of consequence.

To demonstrate this further, let us take the data from Table 7 of the paper by Walker, Bloem and Mullen.[2] Since the data are not readily amendable to examination for the DFE–Δc relation, Table C has been prepared from their Table 7. The DFE's shown have been computed or estimated as DFE at 100 cycles from the following relation: $DFE = PN/100$, where P is the relative dynamic E at N cycles, if 50 or above, or is 50 if the relative dynamic E has fallen below 50. N is 100 cycles if $P > 50$, or the number of cycles at which $P = 50$.

Upon plotting the DFE's against Δc for these data (Fig. A) it is readily apparent that in all six cases, for the same coarse aggregate tested with different sands, the lower the Δc the higher the DFE. This is in accordance with the results of the paper. The author feels that there is the relation between DFE and Δc in these data. However, both for these and for other data there is often a masking action due to variations in aggregate quality, particularly when various types of coarse aggregates are used. To show this, the data of Table D are presented.

These data are the results of the laboratory phase of a program wherein six coarse aggregates were tested with each of eight fine aggregates. All the aggregates represented a wide range of mineralogic composition and thermal coefficients, ranging from low coefficient limestone to high coefficient quartzite,

TABLE C

Fine aggregate	Coarse aggregate	Δ c	Cycle 3				Cycle 6A
			No. of cycles	Dynamic *E* % of orig.	No. cycles = 50%(est) *DFE*		*DFE*
Quartz sand	Quartz	−0.5	48	48	46	24	94
Mfd. limestone	Quartz	−2.7	87	53	93	47	102
Quartz sand	Limestone	3.4	86	53	91	46	94
Mfd. limestone	Limestone	1.2	98	72	—	72	103
Quartz sand	Trap rock	1.9	65	47	61	31	89
Mfd. limestone	Trap rock	−0.3	98	64	—	64	103

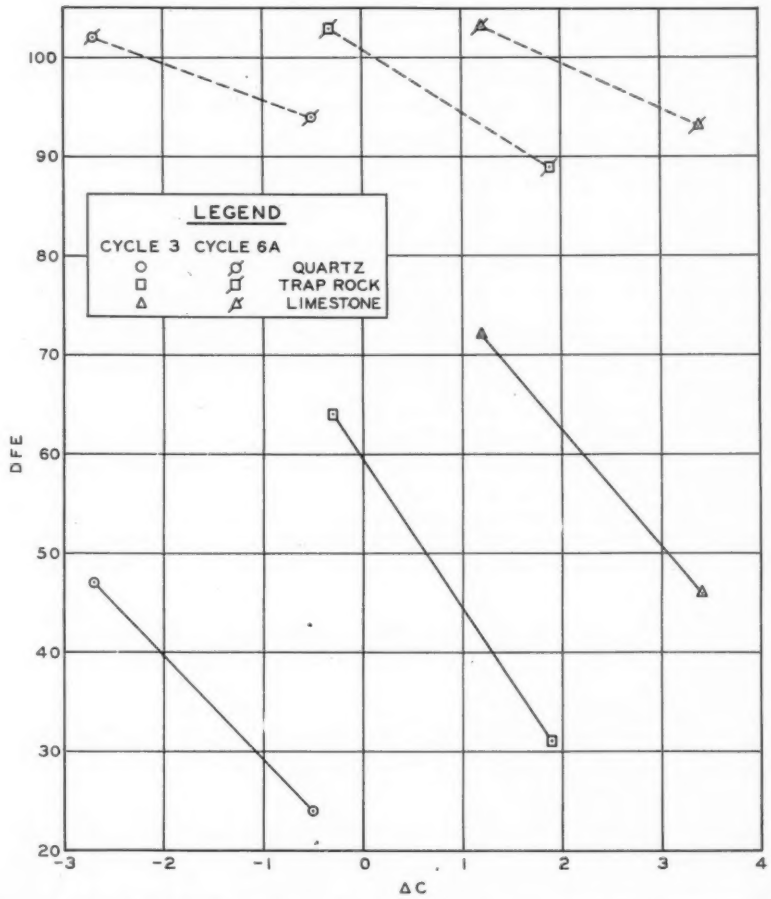

Fig. A—Relation of *DFE* to Δc for six concrete combinations exposed to heating and cooling in water from 40 to 140 F. *DFE* based on 100 cycles; Cycle 3—rapid; Cycle 6A—slow

and including surface texture variations between crushed and uncrushed cherts. When freezing and thawing test results in terms of *DFE* are plotted against thermal coefficient differences in terms of Δc (Fig. B), the relation $DFE = 64.80 - 2.91\ \Delta c$ is found, having a correlation coefficient $r = 0.312$. This is not a high correlation as it stands. To afford a better comparison of the data, the effects of variation in aggregate quality, *e.g.*, the superiority of crushed chert over uncrushed chert, were removed by a somewhat intricate weighting of the observed *DFE*'s, the procedure for which is rather too detailed for description here. The adjusted *DFE*'s are as shown in Table E. Upon plotting these against Δc (Fig. C) the relation $DFE = 66.33 - 6.97\ \Delta c$ is found, with a correlation coefficient of $r = 0.847$.

These results should be construed only as a general indication that the $DFE-\Delta c$ relationship holds for these combinations, although hidden in the actual results due to the effects of other factors affecting the over-all concrete quality. As an example of such factors, it is seen that although the quality of the granite coarse aggregate is quite high, the quality of the sand manufactured from it is quite low. This is probably due to the type of fracture incurred in crushing, leaving the larger sand size particles in a weakened condition. Thus the concretes made with granite coarse aggregate have good *DFE*'s in spite of the large Δc, whereas those made with the manufactured granite sand show lower *DFE*'s than would be expected. Similarly, the surface texture effects of uncrushed chert lead to lower *DFE*'s than when it is crushed and the bond improved, even though the Δc for each comparative combination is the same whether crushed or uncrushed chert is used.

The author is in agreement with Messrs. Willis and Reagel that the combinations included in the paper did not represent a most desirable selection for definitely establishing the $DFE-\Delta c$ relationship. Data were not available for a proper sampling at the time the paper was prepared. It is realized that the ambiguous expectations in respect to *DFE* values pointed out in their table exist. It was felt, however that the benefits to be gained by a discussion of the problem and the focusing of attention on the question would

TABLE D—FREEZING AND THAWING TEST RESULTS OF 48 CONCRETE COMBINATIONS OF VARIOUS THERMAL PROPERTIES

Coarse aggregate	Fine aggregate	*DFE* of concrete combination*							
		Limestone	Granite	Siliceous river	Non-chert sand	Natural siliceous	Cherty river	Quartzite	Chert
	Coeff. of mortar†	4.6	5.7	6.3	6.4	7.1	7.3	7.4	7.5
	Coeff. of aggregate†								
Limestone	2.3	50	30	28	33	23	56	34	28
Granite	2.6	80	64	66	68	78	75	80	72
Non-chert gravel	4.7	46	46	38	62	40	62	35	56
Uncrushed chert	5.9	74	45	40	45	64	56	48	43
Crushed chert	5.9	88	66	71	77	78	89	66	80
Quartzite	6.4	61	59	82	75	85	63	60	70

**DFE* after 300 cycles of accelerated laboratory freezing and thawing.
†Coefficient of thermal expansion $\times 10^6$ per deg F.

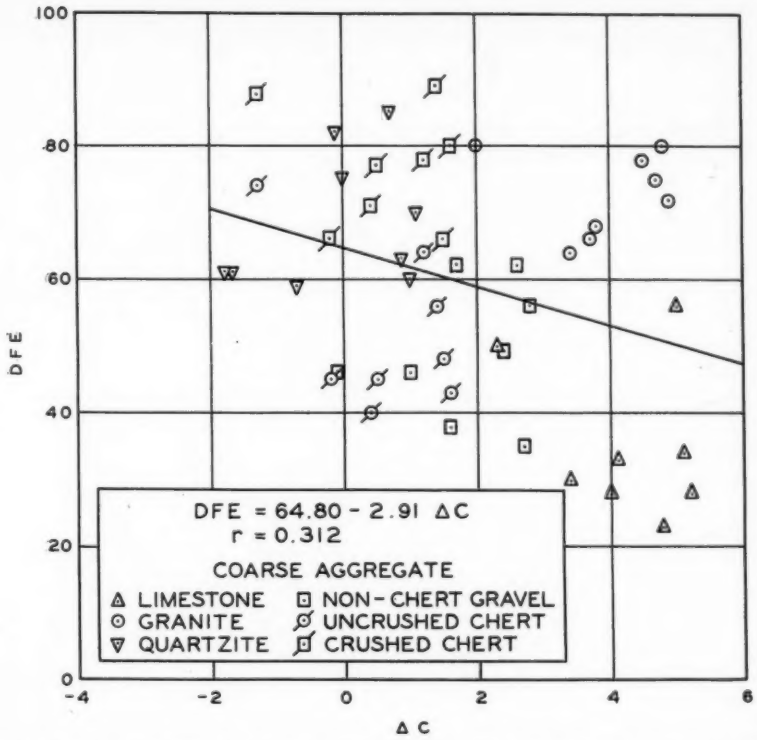

Fig. B—Relation of *DFE*–Δc for 48 concrete combinations. *DFE* obtained from accelerated laboratory freezing and thawing tests; Δc the difference in coefficients of thermal expansion of mortar and coarse aggregate

TABLE E—ADJUSTED *DFE*'s OF 48 CONCRETE COMBINATIONS OF VARIOUS THERMAL PROPERTIES

Coarse aggregate	Fine aggregate	Adjusted *DFE* of combination*							
		Mfg. limestone	Mfg. granite	Siliceous river sand	Non-chert sand	Natural siliceous sand	Cherty river sand	Mfg. quartzite	Chert sand
	Coeff. of mortar†	4.6	5.7	6.3	6.4	7.1	7.3	7.4	7.5
	Coeff. of aggregate†								
Limestone	2.3	57	45	35	33	18	43	34	23
Granite	2.6	52	44	38	28	38	27	45	32
Non-chert gravel	4.7	61	69	53	70	52	57	43	59
Uncrushed chert	5.9	90	69	56	54	68	52	57	47
Crushed chert	5.9	78	64	61	60	56	59	49	58
Quartzite	6.4	62	68	83	69	74	44	54	59

**DFE* of Table D adjusted for variations in aggregate quality.
†Coefficient of thermal expansion × 10^6 per deg F.

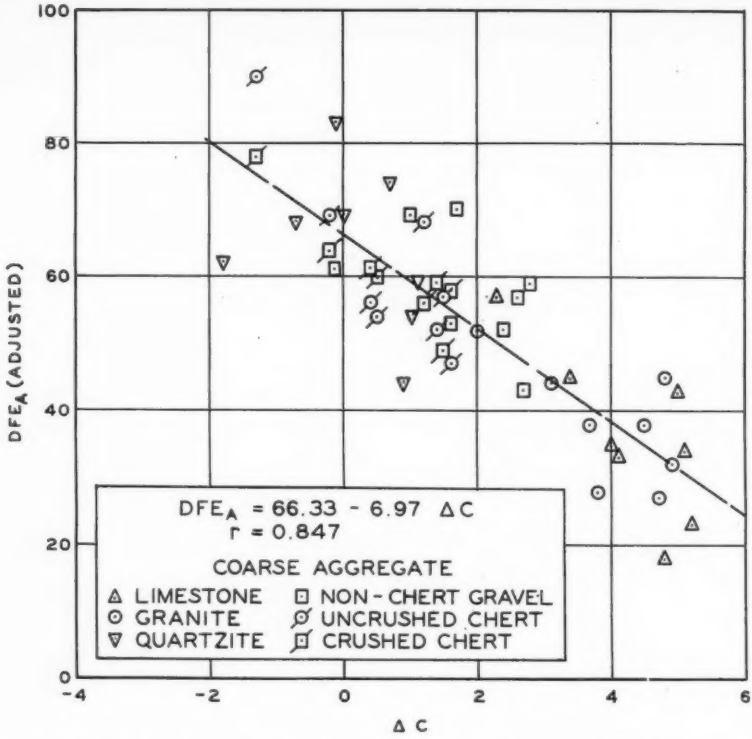

Fig. C—Relation of adjusted DFE_A–Δc for 48 concrete combinations. DFE_A adjusted from freezing and thawing test results for variation in aggregate quality

outweigh the incompleteness of the sampling scheme and the ambiguity in the possible reasons for decreased durability.

The author is in agreement with both Messrs. Willis and Reagel, and Bloem and Walker that the correlations between DFE and c_m (coefficient of mortar) and between DFE and Δc are of about equal statistical significance. It should be noted, however, that except for the change in concrete coefficient occasioned by the different mortars, affecting the stresses as described previously, this correlation of DFE and c_m would rest on no physical basis, whereas that between DFE and Δc would. Considering the stress difference due to coefficient of concrete difference, which amounted to only about 60 psi at ½ in. from the surface, it is not believed that the variations in DFE would be explainable on this basis to any marked extent. It is true that, as shown in "Phase I" of Messrs. Willis and Reagel's discussion, the DFE–Δc relationship can neither be conclusively proved nor discarded on the statistical basis

alone. It is also true that the combination of the statistical basis and physical reasoning leads to placing greater credence on the $DFE-\Delta c$ relationship than on the $DFE-c_m$ relation as affecting concrete durability.

The author is grateful to Messrs. Willis and Reagel for forcefully pointing out that the relation developed should not be considered blindly to hold for other concretes. This caution appears not to have been made as strenuously as need be in the paper and misunderstandings may have arisen from this. The question raised in "Phase III" as to the correlation between laboratory and field results is also of great importance. For the case of the 48 combinations mentioned in this closure, the results to date of the laboratory and field tests on these combinations are reported by Kennedy.[8]

The author feels that these discussions have helped considerably in clarifying the limitations of the $DFE-\Delta c$ relationship, and in bringing attention to bear on the many unsolved facets of the over-all effects of thermal properties on concrete durability. In closing, the author would like to present a quotation from ten years ago, which is still highly appropriate.[9]

"The results obtained to date have indicated the necessity for further study beyond the original scope of the program. Greater knowledge is needed of the thermal properties of hardened concrete and its ingredients, and the internal stresses created in concrete by thermal shock. The interesting prospect is presented that durability of concrete may not be a simple function of the inherent soundness of cement or of the inherent soundness of aggregate, but rather a function of the ability of a given cement to develop a hardened product whose properties are and remain compatible with those of the aggregate used. It seems possible that the use of one cement-aggregate combination might result in concrete of poor durability; whereas, that cement and that aggregate used with another aggregate and another cement might result in concrete of good durability. This possibility may explain some of the confusing paradoxes in concrete durability."

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Title No. 48-34

Relative Economy of Prestressed and Conventional Reinforced Concrete Reservoirs*

By G. C. ERNST†, C. O. BRUNKEN‡ and A. R. RIVELAND‡

SYNOPSIS

Cylindrical walls designed for various stress combinations for conventional and prestressed conditions are compared, and the most advantageous are selected for an economy study. Quantities of concrete and steel from 180 complete reservoirs and 80 additional wall designs reveal that those of prestressed concrete have a lower first cost than conventional designs, if the unit cost of the prestressed wall in place does not exceed approximately twice that for the conventional wall. The investigation also showed that in most cases of unrestricted site location with the same controlling water elevation, reservoirs above ground were more economical than underground reservoirs, insofar as first cost is concerned. However, underground reservoirs could compete with those above ground for capacities greater than 2.5 million gallons. As to materials, prestressed construction saved from 40 to 60 percent of steel and from 0 to 35 percent of concrete.

INTRODUCTION

The need to prepare alternate designs for a 5 million gallon water storage reservoir at three different site locations provided an opportunity to study the relative economy of prestressed and conventional designs. As a preparatory step it was necessary to compare tentative wall designs and select a practicable combination for each type reservoir of the required capacity.

The effect of unit wall costs were then studied, followed by a comparison of first costs of underground and aboveground storage for the specific sites under consideration; and finally the relative savings in material were reviewed for the most competitive types. The order of competition as to first cost was determined and tabulated against the various design types and locations (Fig. 1). It is recognized that with different site conditions the relationships may change but the relative comparisons of aboveground tanks should apply in any situation.

SELECTION OF STRESSES

Several different stress combinations have been used in the past, and have

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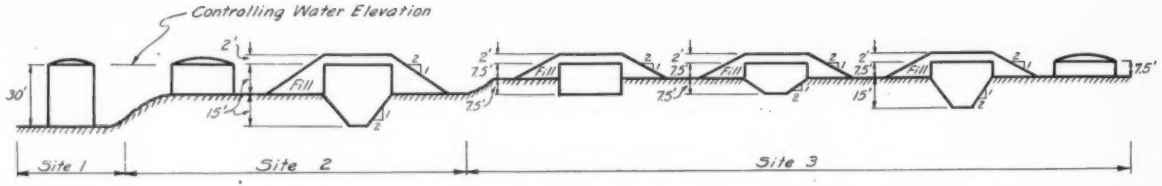


Type		A	B	C	D	E	F	G
Cylindr.	Pre-Stress	1	1	1	5	3	2	2 (1 MG or less)
	Conventional	1	1	1	5	3	2	2 (1 MG or less)
Rectangular				4	6	5	3	

Common Excavation at All Sites
Compacted Embankment at All Sites
Capacities from 0.2 MG to 6 MG
Domed Roofs on All Tanks Above Ground
Slab, Beam, & Column Roofs on Underground Tanks
Same Rating for Different Types Indicates Competitive Costs for Most Capacities, but not necessarily all.

	Stresses					
	f_s	f'_c	f_{si}	f_{ci}	f_t	n
Prestressed	100,000	3000	140,000	1000	—	9
Conventional	14,000	3000	—	—	300	10
Rectangular (cantilever)	20,000	3000	—	—	—	10

Fig. 1—Reservoir types and order of competition as to first cost

TABLE 1—STRESS COMBINATIONS USED IN ECONOMY STUDY

Conventional design				Prestressed design			
Factor	Group a	Group b	Group c	Factor	Group a'	Group b'	Group c'
f_s, psi	10,000	12,000	14,000	f_s, psi	12,000	40,000	100,000
f_t, psi	150	200	300	f_{si}, psi*	45,000	75,000	140,000
n	15	10	10	f_{ci}, psi*	1,000	1,000	1,000
				n	10	10	9

*f_{si} and f_{ci} are for the initial prestressed condition

had their advocates, for both conventional and prestressed design of cylindrical tanks.

The combinations selected for study (Table 1) included low, intermediate and high values.

To determine the design possibilities of each of these stress groups, unrestrained walls were designed for 10 different capacities from 0.2 to 6 million gallons for various wall heights using each of the stress groups in Table 1. It was soon noted that the maximum amount of steel for convenient placement was reached at lower wall heights for some stress groups than for others. Fig. 2 shows the maximum wall heights for practicable steel placement for each stress group. Note that stress groups c (conventional) and b' (prestressed) are about equally competitive on such a basis, and that stress group c' has no limitations within the boundaries of Fig. 2.

The stress groups c and c' were selected for the remainder of the study because they permitted maximum wall heights in conventional and prestressed designs. In other words, the combinations c and c' were the most competitive stress groups of all of the conventional and prestress combinations.

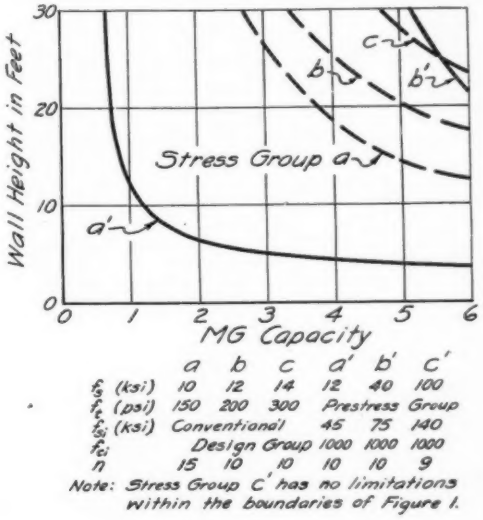


Fig. 2—Maximum heights for cylindrical walls, based on practicable steel placement

ECONOMY OF PRESTRESSED TANKS

Since the walls are only one portion of a tank, changes in the cost of the walls are not reflected in equal degree in the final total cost of the tank. The final total cost of tanks above the ground can be expressed as $C_t = C_w + C_r + C_f$ with C_t, C_w, C_r and C_f representing the total final cost of tank, wall, roof and floor, respectively, provided that the incidental costs of foundation preparation, etc., are included. For underground reservoirs, the total costs of excavation and compacted enbankment must be added.

Quantities of materials were computed in detail for 10 different capacity tanks, 0.2 to 6 million gallons, for each of two heights, 15 ft and 30 ft, for conventional cylindrical and, 7.5 ft and 15 ft, for cantilevered wall rectangular reservoirs. These quantities were multiplied by unit prices, generally prevailing in 1950–51, in the area in which the 5 million gallon tank, previously mentioned, was to be built. The resulting average unit cost of the reinforced concrete in place in the wall was \$74 per cu yd. For comparison three different unit wall costs were selected; \$50, \$75 and \$100 per cu yd for conventional designs. Using the same procedure for obtaining an average unit wall cost for the prestressed tanks, and selecting a low value somewhat in excess of that for conventional design and a high value almost double the maximum value of \$100 per cu yd for conventional design, resulted in the average unit wall cost of \$55, \$125 and \$195 per cu yd for prestressed reinforced concrete in place.

With unit costs for reinforced concrete in place, the formula previously expressed becomes $C_t = U_w V_w + U_r V_r + U_f V_f$ in which U_w, U_r and U_f are unit costs of reinforced concrete in place in wall, roof and floor, respectively, in dollars per cu yd, and V_w, V_r and V_f are cu yd of reinforced concrete in wall, roof and floor, respectively.

It is readily recognized from the above equation that, with virtually fixed values for U_r, V_r and U_f and V_f for a specific tank, variations in either U_w or V_w or both (by use of prestressed concrete, or a change in unit stress combination) will be indicated in lesser degree in the total price C_t.

If comparisons of the cost are made between prestressed and conventional designs, the cost ratio is as follows, the primed values referring to the prestressed design:

$$\frac{C_t'}{C_t} = \frac{U_w'}{U_w}\left[\frac{V_w' + 1/U_w'\,(U_r'V_r' + U_f'V_f')}{V_w + 1/U_w\,(U_r V_r + U_f V_f)}\right]$$

also

$$\frac{C_t'}{C_t} = \frac{C_w' + C_r' + C_f'}{C_w + C_r + C_f}$$

If the values of U_w, U_w', V_w and V_w' are such as to make $C_w' = C_w$ the value of C'_t/C_t will be slightly less than one, since $C_r' + C_f'$ is less than $C_r + C_f$ due to the slight reduction in outside dimensions when prestressed walls are used. Therefore, theoretically, the unit price of prestressed reinforced concrete in place in the wall can be slightly higher for equal total tank costs

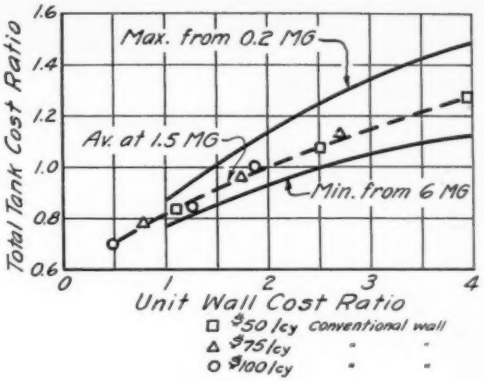


Fig. 3—Variation of unit wall cost with total cost. Prestress tank (type B) *vs* conventional (type B). Each point equals average of ten capacities

than for equal total wall costs. In this study, the effect was quite evident in capacities below 1 million gallons, although the effect from 1 to 6 million gallons was practically imperceptible.

The unit wall costs previously stated were used in calculating unit wall cost ratios and total tank cost ratios between prestressed and conventional designs for both 15 and 30 ft heights, with capacities from 0.2 to 6 million gallons. Unit costs for roof and floor were taken at average 1950–51 values and were kept the same for all designs. A study of the unit wall and total tank cost ratios showed that the average of all capacities from 0.2 to 6 million gallons in each unit cost group plotted practically on the same curve, and corresponded to a capacity of 1.5 million gallons (Figs. 3 and 4). Therefore total tank costs for prestressed and conventional tanks become equal for a

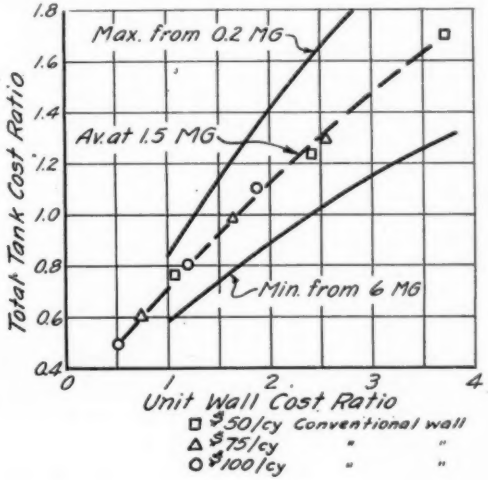


Fig. 4—Variation of unit wall cost with total cost. Prestress tank (type A) *vs* conventional (type A). Each point average of ten capacities

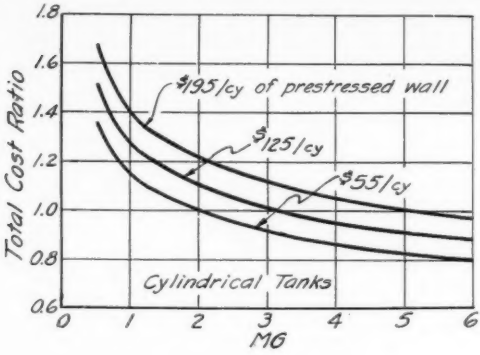


Fig. 5—Variation of total cost with tank volume. Underground prestressed tank (type C) *vs* conventional tank (type A)

capacity of 1.5 million gallons when the unit wall cost for prestressed construction is twice that for conventional construction for the 15 ft height, and 1.7 for the 30 ft height. Upper values of the total cost ratio were consistently defined by the 0.2 million gallon capacities and the lowest values of the ratio were consistently defined by the 6 million gallon capacities. This does not mean, however, that all total cost ratios for 0.2 million gallons or for 6 million gallons fall on the upper and lower lines shown in Fig. 3 and 4. However, the solid lines (Fig. 3 and 4) define the maximum spread of all total cost ratios above and below the average. These lines show a range of unit wall cost ratios of from 1.5 to 2.7 for 15-ft tanks and 1.3 to 2.4 for 30-ft tanks for equality of total costs.

COST OF UNDERGROUND AND ABOVEGROUND RESERVOIRS COMPARED

Underground reservoirs, due to the added cost of excavation and embankment, are generally more expensive than reservoirs above the ground. Requirements other than first cost usually dictate their selection in a specific case. However, in the case of capacities in excess of about 3 million gallons underground tanks with prestressed walls and imbedded floors may become competitive with conventional aboveground tanks and with prestressed tanks of shallow height, for sites permitting common excavation (Fig. 5 and 1).

ECONOMY OF MATERIALS

Prestressed designs show savings of 52 to 61 percent in weight of steel and 18 to 35 percent in volume of concrete for cylindrical reservoirs 30 ft high (Fig. 6). Comparison of prestressed underground reservoirs with conventional aboveground tanks 30 ft high showed possible savings of 42 to 57 percent in steel and −12 to +30 percent in concrete, providing the base of the underground tank is imbedded (Fig. 6). With the exception of the steel in the prestressed tanks above the ground, the saving in material increased with the capacity of the tank, for a given height requirement.

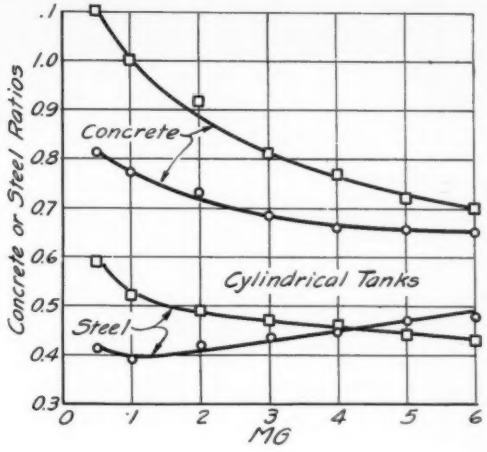


Fig. 6—Prestressed steel and concrete savings. ○ Prestressed tank (type A) vs conventional (type A). □ Underground prestressed type C) vs conventional (type A)

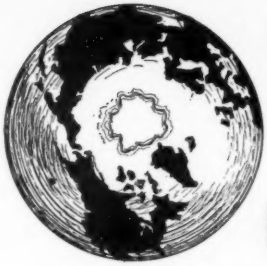
SUMMARY

1. The stress combinations designated as group *c* (conventional) and group *c'* (prestress) were found to be the most satisfactory combinations in designing tanks through a range of capacities from 0.2 to 6 million gallons, and up to 30 ft in height.

2. In general, if the unit price of prestressed wall concrete in place is less than twice the unit price for conventional design, a prestressed tank may be expected to have a lower first cost than one of conventional design, especially in the case of above the ground tanks of about 1.5 million gallon capacity.

3. With ideal excavation and price conditions, underground tanks may have a lower first cost in capacities greater than 3 million gallons, if designed with a relatively deep floor imbedment.

4. Substantial savings in material, up to 61 percent in steel and 35 percent in concrete, may be expected in prestressed aboveground tanks. Underground tanks having a capacity in excess of about 1 million gallons in general showed savings, but to a lesser degree.



Letters from Readers*

BY WAY OF SYNOPSIS

W. C. HANNA adds further information on the inhibition of alkali-aggregate expansion.

WILBUR E. MOULTON discusses water content of pumice concrete.

JEAN LEBELLE tells of special vibrating equipment developed for use with no-slump concrete on Creil Airport in France.

HERMANN CRAEMER explains a design method for skew bending in reinforced concrete.

TAYLOR WOODROW CONSTRUCTION, LTD., gives data on the prestressed concrete footbridge built across the River Severn in England.

Additional Information on Inhibiting Alkali-Aggregate Expansion (LR 48-19)

The results of tests on inhibiting alkali-aggregate expansion reported by W. J. McCoy and A. G. Caldwell† are of much interest. It is hoped that additional information is available which will help answer some questions.

It has been well-known for a number of years that additions to reactive combinations of cements and aggregates have not always produced the results expected. Each particular aggregate, cement, proportion, addition, and curing cycle has an effect. In some cases small additions such as *NaOH* have been expected to produce increased expansion, but results have indicated almost no expansion. In some cases the addition of sodium silicate has lowered expansions but lowering expansion is not in itself sufficient. It is hoped that strength tests on the promising combinations mentioned will be published.

Referring to Table 12, can the authors give results with both greater and lesser amounts of Li_2CO_3 added and also in combinations with other cements? Do they have results with a low-cost chemical such as *NaOH*?

Referring to the third paragraph on page 695 of their paper, the writer has seen structures in service in which low-alkali cement was reported to have been used which showed reactive expansion. Other structures appear in good condition although containing potentially reactive combinations but which had not been exposed to destructive cycles. The actual alkali content in most old cements probably was not known and bad work (since it was bad) has been attributed to high-alkali cement or if it was good to low-alkali cement. Such careless assumptions are hardly acceptable.

W. C. HANNA, chief chemist and chemical engineer, Calif. Portland Cement Co., Colton, Calif.

*A part of copyrighted JOURNAL OF THE AMERICAN CONCRETE INSTITUTE, V. 23, No. 6, Feb. 1952, *Proceedings*, V. 48. Separate prints are available at 35 cents each. Address 18263 W. McNichols Rd., Detroit 19, Mich.

†McCoy, W. J. and Caldwell, A. G., "New Approach to Inhibiting Alkali-Aggregate Expansion," ACI JOURNAL, May 1951, *Proc.* V. 47, p. 693.

Water Content in Pumice Concrete (LR 48-20)

J. A. Murlin* states that "the slump cone test is a poor indicator of water content or of workability of lightweight concrete . . . " I am in complete agreement with this statement because of our experiences with the water content in pumice concrete.

Mixing time, workability, and whether or not material has been pre-saturated alter the required amount of water.

We do not pre-saturate our material; so more water must be used. A minimum of 12 minutes mixing time is essential; 15 minutes is still better.

In a given mix we have used from 57 to 63 gallons of water per cu yd and the slump has been the same. When the slump cone is lifted, the material is left standing upright with little or no slump.

If there is sufficient water, the aggregate and cement will hold together in a sticky mass as it is thrown up and down in the hand. A dry mix has a crumbly feel and appearance. If too wet, the mix is sloppy or swimming in water.

After the material is placed and vibrated, a mix with sufficient water will finish well. In an overwet mix the coarse material rises to the top, causing a rough finish. A too-dry mix will not vibrate into place or hold together well.

So far, experience with pumice concrete is the only practical way to determine the proper amount of water to be used.

WILBUR E. MOULTON, testing engineer, Normac, Inc., Monterey, Calif.

Vibration of Pavement Concrete (LR 48-21)

The ACI Journal, Mar. 1950, contained a very interesting paper by F. H. Jackson† pointing out the disappointments met in the construction of concrete roads in the United States by the use of too plastic mixes.

In the discussion which followed, different persons referred to work executed in Europe, and Phillip L. Melville stated the results obtained on our Auto-route de l'Ouest.

For more than 20 years, the writer has devoted his activities to the development of powerful vibration equipment called "Pervibrateurs" which are used on the majority of construction jobs in France and are known in many foreign countries. It is this equipment which has permitted the use of gap-graded concrete with very low water content.

Recently we have been consulted on the runways for Creil Airport constructed of slabs 6.25 m (20.5 ft) wide and 25 cm (9.84 in.) thick. The conditions imposed by the specifications were particularly rigorous. The mixing water, reduced to the minimum, is determined as follows:

45 l (11.89 gal.) of water plus 0.25 l per kilogram (0.03 gal. per lb) of cement. The prescribed cement content being 350 kg (770 lb) of cement, the quantity is then: $45 + (350 \times 0.25) = 132$ l (35 gal.)

This quantity of water is imperative regardless of the nature of the cement and the quality of the aggregates. It takes into account the water contained in the aggregates. This always results in a very dry concrete, very difficult to place, and which cannot be considered workable without powerful vibrators.

On the Creil work a gap grading was adopted, the principal characteristics of which are given below.

There is no material provided between the coarse sand of 6.3 mm (0.25 in.) and the small stone of 31.6 mm (1.25 in.) which represents, conforming to the rule, a discontinuity of 1/5.

The maximum diameter of gravel is 80 mm (3.15 in.), represented by the rounded flint particles.

It is not possible to speak of slump because of the use of aggregates approaching 80 mm but if the test were made it would give a zero slump.

For this mix, the following materials were used:

*Murlin, A. J., "Lightweight Concrete for Lower Construction Costs," ACI JOURNAL, Sept. 1951, *Proc.* V. 48, p. 37.

†Jackson, F. H., "A Way to Better Pavement Concrete," ACI JOURNAL, Mar. 1950, *Proc.* V. 46, pp. 489-496.

Fig. 1—Construction of Creil Airport runway. Concrete placed by Blaw-Knox highway equipment

Gravel	1000 l (35.3 cu ft) (Apparent) or 520 l (18.4 cu ft) (Absolute)
Sand	395 l (13.9 cu ft) (Apparent) or 235 l (8.3 cu ft) (Absolute)
Cement	350 kg (770 lb)
Water	132 l (35 gal.)

Thus $w/c = 0.37$

The concrete was mixed in a horizontal axis mixer with an actual volume of 800 l (28.25 cu ft). It is to be noted that the dry mixes made in these mixers are difficult to discharge. The central mixing plant was one to two kilometers from the work.

The concrete was carried from the central mixing plant in large cylindrical containers transported by truck to the job where the container was lifted by a crane and dumped in the pavement forms. The floating and placement was done with a Blaw-Knox spreader (Fig. 1).

In the first tests, it was evident that the Blaw-Knox machine designed for placing plastic American concrete could not assure the compaction of concretes conforming to the French specifications and, in similar work done previously, it had been found necessary to construct the pavement in two layers laid one after the other, without an appreciable time interval.

For Creil Airport, the Blaw-Knox Co. of France had requested us to modify its machine to augment the force of vibration.

We have replaced the vibrating beam by separate elements, of appropriate form and weight, each equipped with an electric vibrator. A motor-generator of sufficient capacity to drive all the vibrators was installed on the machine.

Because of the difficulty in supplying concrete, construction speed was limited to 20 to 30 cu m (706 to 1059 cu ft) per hour. The machine would have permitted an output 4 or 5 times as great.

It has been verified from cores drilled out

Fig. 2—Highway machinery with modified Blaw-Knox vibrating equipment

of the finished pavement that the compaction was absolutely complete throughout the 25 cm (9.84 in.) thickness. Later tests showed that one can consolidate satisfactorily pavements up to 35 cm (13.8 in.) thick.

With very dry mixes, there is little bleeding and large aggregates are abundant near the surface. This is, after all, one of the objectives it is proposed to obtain.

Then using the finisher which is usually employed in the United States for finishing pavements and which contributes to bleeding of mortar, the surface was torn, and the stones are found to be dragged apart by the passage of the finishing element. A small change permits remedying this inconvenience and to obtain a surface conforming to the specifications. One can verify on Fig. 2 that the surface obtained by the spreader equipped with vibrators before the passage of the finishing machine is nearly satisfactory, the unevenness on a section 6.25 m (20.5 ft) long scarcely exceeding 1 cm (0.4 in.).

Following the results obtained on Creil Airport, we have ordered similar equipment for Rabat (Sale) Airport in Morocco, the specifications being similar to those indicated above.

JEAN LEBELLE, President,
General Director, Procédés
Techniques de Construction
Paris, France

Skew Bending in Reinforced Concrete Computed by Plasticity (LR 48-22)

If a symmetrical reinforced concrete cross section is acted upon by two bending moments, one turning around a horizontal axis and the other about a vertical one, it is said to be subject to double or skew bending. The several conventional design methods for this case assume that both steel and concrete behave elastically and thus make use of the modular ratio. However, they are so cumbersome that they are not frequently used.

In the theory of plasticity, however, great simplification is possible by assuming an *equal* distribution of the compressive stresses. In this problem the results of Professor Saliger's research work are adopted. He found that, at failure, the stresses in a beam under simple bending are distributed as in Fig. 1, the stress at the top being equal to the prism strength, f_c', the average stress being about $0.85\,f_c'$. He further showed that with the exception of beams with *too* heavy reinforcement, which should not be used in practice, the steel is stressed with the yield stress, f_y. In the following it is assumed that the resultant of the compressive zone is situated in its centroid, thus simplifying Saliger's theory which gives a more precise but only slightly more favorable value.

Skew bending without normal force

Let a cross section be acted upon by two bending moments, M_h' and M_v' (the dashes in the figures symbolize the failure stage) turning about a horizontal and a vertical axis, respectively. Then the neutral axis will be inclined and pass through either two opposite or two adjacent sides of the rectangle (Fig. 2 and 3). In Fig. 2 the centroid of the area, A_s, of the tensile reinforcement is supposed to have a distance e from the center line. With fair accuracy we may use Saliger's results (average stress $0.85\,f_c'$, resultant in centroid of area), *i.e.*, consider the area $ABCD$ to be equally stressed by $0.85\,f_c'$. The compressive reinforcement shown will be considered later.

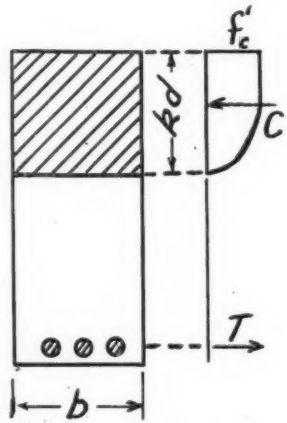


Fig. 1

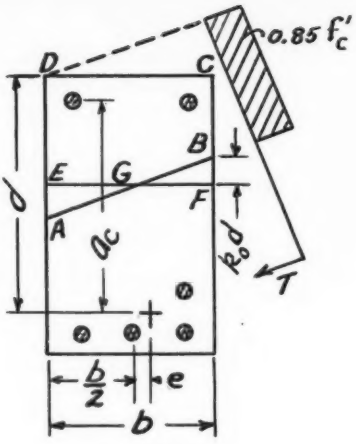


Fig. 2

Then the whole compressive force is

$C = 0.85\, f_c'\, kd\, b$

and, from the equilibrium of forces, we get

$O = 0.85\, f_c'\, kd\, b - A_s f_y$ ..... (1)

In the following consider the trapezoid $ABCD$ to be composed of the rectangle $EFCD$ plus and minus, respectively, the triangles EAG and BGF, the compressive force in each of the latter being $+ 0.85\, f_c'\, b\, kd/4$. Taking the moment about the horizontal axis through the tensile force we thus have

$M_h' = 0.85 f_c' b\left[kd\,(d - \tfrac{1}{2}\,kd) - \tfrac{2}{6}\,(k_o d)^2\right]$ (2)

and for a vertical axis through the same point

$M_v' = 0.85\, f_c'\, b\,(kd\, e + \tfrac{1}{6}\, b\, k_o d)$ .... (3)

Now, in the working range, we write M_h and M_v instead of M_h' and M_v' and use the allowable stresses, f_c and f_s instead of the ultimate, f_c' and f_y. An increase in the allowable stress, which in some regulations is provided for the case of skew bending, should not be taken into account since this allowance is meant to represent approximately the favorable influence of plasticity which is of greater importance in skew bending than simple bending. Eq. (1) to (3) can be transformed into

$$A_s = \frac{0.85\, f_c\, kd\, b}{f_s} \quad \ldots\ldots (4)$$

$$kd = \frac{1}{1 - \dfrac{kd}{2d}}\left(\frac{M_h}{0.85\, f_c\, bd} - \frac{(k_o d)^2}{6d}\right) \quad \ldots (5)$$

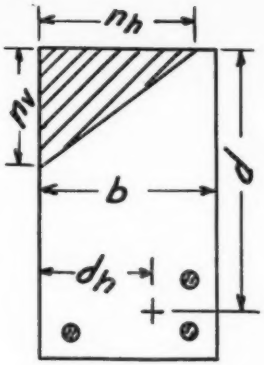


Fig. 3

$$k_o d = \frac{6\, M_v}{0.85\, f_c b^2} - \frac{6e}{b}\, kd \quad \ldots\ldots (6)$$

This form is most suitable for solution by iteration (repetition) as is shown in the following.

Example 1

Let a beam having $b = 12$ in., $d = 24$ in., $e = \frac{1}{2}$ in., $f_c = 850$ psi, and $f_s = 18{,}000$ psi be subjected to $M_h = 2{,}000{,}000$ in.-lb and $M_v = 100{,}000$ in.-lb. Find the necessary reinforcement, A_s. By inserting these values in Eq. (5) and (6) we obtain

$$kd = \frac{1}{1 - 0.021\, kd}\,(9.60 - 0.0069(k_o d)^2)$$

and

$$k_o d = 5.77 - 0.25\, kd$$

Now, assuming $kd = 12$ in. as a starting approximate value and inserting it into Eq. (2) we get 2.8 as a first approximation for $k_o d$. Inserting this and $kd = 12$ into the first equation we find an improved value for kd and, proceeding in the same manner the following series is easily gotten by slide rule

$kd = 12 \quad 12.8 \quad 13.1 \quad \textit{13.1}$

$k_o d = \quad 2.8 \quad 2.6 \quad \textit{2.5}$

It is seen that the neutral axis actually corresponds to the pattern of Fig. 2. From Eq. (4) the reinforcement is $A_s = 6.3$ sq in.

If the neutral axis intersects two adjacent sides (Fig. 3) the bending forces are, when the ultimate stresses are replaced by the working stresses, $C = \frac{1}{2}\,(0.85\, f_c)\, n_h n_v$ and $T = f_s A_s$, thus

$A_s = 0.85\, f_c\, n_h n_v / 2 f_s$ ..... (7)

and, from the moment about the centroid of the reinforcement

$$M_h = C\ (d - \tfrac{1}{3}\ n_v)$$

therefore

$$n_h = \frac{1}{1 - n_v/3d} \cdot \frac{2M_h}{0.85\ f_c\ n_v d} \quad \ldots\ldots\ldots\ (8)$$

and correspondingly

$$n_v = \frac{1}{1 - n_h/3d_h} \cdot \frac{2M_v}{0.85\ f_c\ n_h d_h} \quad \ldots\ldots\ldots\ (9)$$

Example 2

Let $M_h = 500{,}000$ in.-lb and $d_h = 7$ in., all other data remaining the same as in the first example. We get from Eq. (8) and (9)

$$n_h = \frac{1}{1 - 0.0139\ n_v} \cdot \frac{57.8}{n_v}$$

and

$$n_v = \frac{1}{1 - 0.0475\ n_h} \cdot \frac{39.6}{n_h}$$

and, starting with $n_h = 10$, the following iteration is obtained

$n_h = 10 \quad 8.6 \quad 8.3 \quad \textit{8.2}$

$n_v = \quad 7.5 \quad 7.8 \quad 7.9 \quad \textit{7.9}$

By inserting $n_h = 8.2$, $n_v = 7.9$ into Eq. (7) we obtain $A_s = 0.85 \times 850 \times 8.2 \times 7.9/2 \times 18{,}000 = 1.30$ sq in. It is seen that the values n_h and n_v coincide with Fig. 3.

The reader will observe that the plastic theory leaves him a greater freedom of design since, within reasonable practical limits, the concrete dimensions may be freely chosen and only the reinforcement has to be determined.

Effect of normal force and compressive reinforcement

Let now, at failure, a normal force N' act eccentrically on the cross section, M_h' and M_v' being its moments about the centroid of the reinforcement. In the case of a neutral axis (Fig. 2) we now include the compressive reinforcement, A_C, shown in Fig. 2. Its centroid is assumed to be in the center line.

The theory of plasticity shows that, at failure, the compressive reinforcement is stressed with the yield stress. Thus Eq. (1) becomes

$$N' = 0.85 f_c' kd + A_C f_y - A_s f_y \quad \ldots\ldots (10)$$

and Eq. (2) and (3) become

$$M_h' = 0.85 f_c' b\left[kd\ (d - \tfrac{1}{2}kd) - \tfrac{1}{6}(k_v d)^2\right] + A_C f_y a_c \quad \ldots\ldots (11)$$

and

$$M_v' = 0.85 f_c' b\ (kd\, e + \tfrac{1}{6}k_v d\, b) + A_C f_y e \ldots (12)$$

Passing to the working stage and transforming for iteration, we find

$$A_s = \frac{0.85\ f_c\ kd\ b - N}{f_s} + A_C \quad \ldots\ldots\ldots (13)$$

$$kd = \frac{1}{1 - kd/2d} \cdot \left(\frac{M_h - A_C f_s a_c}{0.85\ f_c\ bd} - \frac{(k_v d)^2}{6d}\right) \quad \ldots\ldots\ldots\ldots (14)$$

and

$$k_v d = \frac{6(M_v - A_C f_s e)}{0.85\ f_c\ b^2} - \frac{6e}{b}\ kd \quad \ldots\ldots\ldots (15)$$

The above equations can also, with $N = 0$, be used in the exceptional case that, under pure bending without normal force, a compressive reinforcement is to be considered.

Example 3

In example 1, assume a normal force $N = 20{,}000$ lb and a compressive reinforcement $A_C = 1.57$ sq in. at distance $a_c = 22.5$ in., all other data remaining the same. From Eq. (14) and (15) we find

$$kd = \frac{1}{1 - 0.021\ kd}\ (6.55 - 0.0069\ k_v d)$$

$$k_v d = 4.95 - 0.25\ kd$$

and by iteration, $kd = 7.8$ in. and $k_v d = 3.0$ in. The tensile area is obtained from Eq. (13): $A_s = (0.85 \times 850 \times 7.8 \times 12 - 20{,}000)/18{,}000 + 1.57 = 4.23$ sq in.

In Fig. 3 the compressive force is comparatively small and a compressive reinforcement, therefore, useless. Thus, Eq. (8) and (9) remain valid and Eq. (7) passes into

$$A_s = \frac{0.85\ f_c\ n_h\ n_v - 2N}{2\ f_s} \quad \ldots\ldots\ldots\ (18)$$

The application is similar to that demonstrated in example 2.

If a normal force acts, the neutral axis may be so near the tensile bars that not all of them reach the yield point. This can be checked as follows. From Saliger's investigations we may conclude that in the most strained corner, A in Fig. 4, the strain at failure is at least 3 per mille. Then, for the bar nearest to the neutral axis the strain will be $0.003x/y$ since the strains are linearly distributed. Now, the yield strain is f_y/E, E being Young's modulus. Thus, the yield stress is reached if

$$0.003x/y \geqq f_y/E \quad \ldots\ldots\ldots\ (19)$$

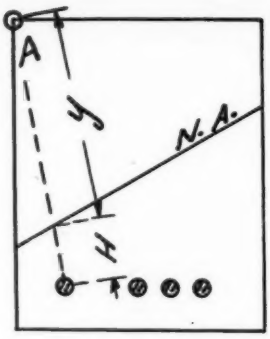


Fig. 4

If the above condition is not fulfilled, the bars are only stressed with $0.003x/y \times E$, *i.e.*, according to Hooke's law. In such a case, instead of the allowable tensile stress, f_s, the term

$$f_s' = 0.003\, E\, f_s / y\, f_y \quad \ldots\ldots\ldots\ldots\ldots\ldots (20)$$

is used in Eq. (13). The position of the neutral axis as obtained by Eq. (8), (9), (14) and (15), however, is not affected by Eq. (20). In the same way, the reduced allowable stress of any other tensile bar can be computed if necessary.

Example 4

Let, in Fig. 4, $x = 2$ in., $y = 8$ in. and a steel with $f_s = 20{,}000$, $f_y = 44{,}000$ and $E = 30{,}000{,}000$ psi be used. Since Eq. (19) is not fulfilled we must use an allowable stress of

$$f_s' = \frac{0.003 \times 2 \times 30{,}000{,}000 \times 20{,}000}{8 \times 44{,}000} =$$

10,200 psi

for the bar in question. If a case like this occurs under bending without normal force, it nearly always means that the cross section is uneconomically designed.

T-Beams

In a T-beam the stiffness of the slab is usually sufficient to carry M_v with negligible stress; only M_h has to be considered in the usual way.

Practical considerations

The increased freedom of design permitted by the theory of plasticity should not be misused. Thus, in pure bending a reinforcements of 2 to 2.5 percent should not be exceeded and a compressive reinforcement is almost always a symptom of careless design. Using less than the yield stress, as discussed in Eq. (19) and (20) should also be avoided. Of course there are exceptions if special reasons require a particularly small height of beam, *e.g.*, due to restricted space or necessity to save weight as in long-span bridges and hangars.

If there is a compressive normal force, however, a compressive reinforcement as well as an understressing of the tensile bars is often unavoidable, especially if the relative eccentricity, M_h/Nd (Fig. 2) is small. The case of Fig. 3, in turn, will only occur if the normal force is comparatively unimportant and then the recommendations for pure bending apply.

HERMANN CRAEMER, Professor of Civil Engineering, University of Alexandria, Egypt

Castle Bridge Across River Severn (LR 48-23)

The footbridge across the River Severn is the result of successful, competitive design and construction against alternative designs submitted in structural steelwork and normal reinforced concrete.

General description

The bridge is formed of two sets of three balanced cantilevers pivoting on 3½ in. diameter steel roller bearings on the original bridge piers, which are at 150 ft centers. A set of four precast 60 ft long girders are suspended on 2½ in. diameter steel roller and rocker bearings at the ends of the cantilevers over the river.

The over-all length of the bridge over the counterpoise block is 247 ft, and the width over the main girders is 11 ft 6 in. The deck slab is formed of 3 in. thick, precast concrete slabs with *in situ* concrete to make up gaps between the precast slabs.

At the piers the beams are 7 ft deep and at the center of the suspended span 2 ft 9 in. deep. The soffit of the bridge is of parabolic arch form with a rise of 7 ft in 150 ft, and

Fig. 1—Castle River Bridge

the deck of the bridge is cambered longitudinally 5.2 ft in 247 ft.

Each of the side cantilevers is 8 ft thick and the center cantilever 16 in. thick. Each set is tied together and stiffened with cross beams at the top at 9 ft centers and at the bottom at 3 ft centers. A projecting flange 4 in. wide at top of the beam carries the precast deck slabs. Each of the suspended girders are of channel section with the web 6½ in. thick, stiffened at 9 ft centers with vertical ribs. The flange of these beams is 12 in. wide, giving a ratio 1/60 of the span. These beams were also tied together at the top with *in situ* concrete beams after erection. Precast concrete beams at 3 ft centers are freely supported between the bottom flanges.

For all the main beams, precast concrete end blocks containing the Freyssinet cones are used.

Bridge piers

The original piers of solid mass concrete supporting the old suspension bridge towers were modified for use in the new bridge.

Counterpoise anchor blocks

These were located between the original suspension bridge anchor blocks and abutment. Both of the new anchor blocks followed normal practice. However, the North block was located in the near middle of Dorset Street, and doubtless caused some inconvenience to local residents, due to the necessary but lengthy time of four months that the excavation had to be kept open.

Precast post-tensioned beams 60 ft long

These were manufactured on the South bank of the Severn, about 100 ft upstream of the new bridge. A cut was excavated in the river bank at right angles to the river, and on the formation which sloped towards the river was laid a concrete slab to carry the new beams during construction. The inclination of the concrete slab served as a slipway down which the beams were moved on 4 in. diameter steel rollers to a timber piled gantry at water's edge. This, with its overhead travelling chain block, served to place the beams on pontoons. The beams were controlled in their movement down the slipway by a pair of crab winches, one pulling, the other retarding the beam, and also by two sets of sway struts. After loading the two sets of pontoons supporting each end of the beam, the whole unit was floated downstream to the lifting position at the bridge, ready for hoisting into their final position.

Prestressing operation

Stressing of the cables was by a pair of Freyssinet jacks, one at each end of the cable. Although a calculated allowance was made for a coefficient of friction of 0.50 between the steel cable and plastic sheath, and also taking into account the effect of curvature in the cable, it was found that friction was higher than calculated, and not consistent. Different results were also obtained for each "half" of the cable. These results tend to suggest that a third effect, *stricture*, not previously considered, takes place.

Under stressing, the main cantilevers lifted their noses about $\frac{3}{16}$ in. and this gap was wedged up before travelling the 16-ton crane over them. The 60 ft long suspended girders lifted a maximum of $\frac{3}{8}$ in. at their center. These members were very highly elastic in their lateral plane, with width to span ratio of 1/60, and received the *respect* they deserved during handling.

Taylor Woodrow Construction Ltd., Ruislip Road, Southall, England

Current Reviews*

of Significant Contributions in Foreign and Domestic Publications

Slag concrete masonry units

National Slag Assn., Washington 4, D. C. 22 pp.

A well illustrated booklet of excellent format describing slag aggregate and the properties of slag-concrete masonry units.

The amount of water in sand (Le dosage de l'eau dans les sables)

M. Houlnick, *Annales des Ponts et Chaussees* (Paris), V. 121, No. 6, Nov.-Dec. 1951, pp. 765-766

Reviewed by Aron L. Mirsky

Drying of sand samples in the laboratory is accomplished quickly by a 250 watt infrared bulb suspended just over the sample. Cost is low: no oven or enclosure is required. Stirring during drying is, however, recommended.

Prestressed concrete—Its future in the U. S.

Michael A. Spronk, *Contractors and Engineers Monthly*, Aug. 1951

Discusses principles of prestressed concrete and its advantages and disadvantages. Reviews basic methods, their application and cites typical structures in which each has been used. Briefly considers American applications of prestressed concrete.

Tests of fine aggregate for organic impurities and compressive strength in mortars

Bryant Mather, ASTM *Bulletin*, No. 178, Dec. 1951, pp. 35-43

Reviews methods used in testing fine aggregate for concrete to determine organic impurities and the relations between the results of color tests, mortar strength tests, and other factors such as soundness and grading. Data are presented on 294 fine aggregates.

Air-entraining concrete in Switzerland (L'Occlusion d'air dans les betons en Suisse)

R. Frey, *Revue des Materiaux de Construction* (Paris), No. 431-432, Aug.-Sept. 1951, pp. 272-276, No. 433, Oct. 1951, pp. 305-308

Reviewed by Phillip L. Melville

Since 1947 Switzerland has followed American practice in entraining air, though some engineers have not yet realized that high densities resulting in high strength may also result in low durabilities. The use of Darex AEA in the construction of Ratherichsboden Dam gave satisfactory results warranting the adoption of air-entrainment by European engineers.

Waterproofing concretes

L. Savoly, *Communication and Civil Engineering in Hungary*, V. 2, No. 11, Nov. 1950, pp. 43-46

Hungarian Technical Abstracts No. 5, 1951

A study of the properties of hydraulic admixtures to concrete which enhance resistance to the passage of water. The article deals with the hypotheses of waterproofness of portland-pozzolan concretes, tests of waterproofing qualities, the connection between the clay content of pozzolans and the behavior of various types of concretes in sulfate waters, blast furnace slags as admixtures, theories of base exchange, and the correlation between shrinking of concrete and watertightness.

*A part of copyrighted Journal of the American Concrete Institute, V. 23, No. 6, Feb. 1952, *Proceedings* V. 48. Address 18263 W. McNichols Rd., Detroit 19, Mich. Copies of articles or books reviewed are **not** available through ACI. In most cases they can be obtained direct from the original publishers. Address, when available, will be furnished by ACI on request.

Concrete cracking Part 2—Pavement studies

L. Boyd Mercer, *Commonwealth Engineer* (Melbourne), V. 39, No. 3, Oct. 1951, pp. 106-112

Discusses the relation of water loss and concrete cracking and considers the factors in evaporation from concrete surfaces. Describes an investigation of cracking in airport runway pavement attributed to variations in the quality of the cement. Concludes that the properties of the cement must be considered in assessing causes of cracking and advocates delayed finishing of concrete.

Abutments for small highway bridges—Part 1

Jacob Feld, *Proceedings*, Highway Research Board, V. 43, 1943, pp. 403-413

A general outline of factors relating to the design of abutments for small highway bridges. Detailed discussion of active earth pressure, passive earth resistance, foundation pressures, bearing values of piles, uplift resistance of piles, lateral and pull-out resistance of soils, lateral resistance of piles, ice pressure and uplift, and scour of water and silt.

Concrete mix design (La composition des betons)

F. Campus, *Cours de Constructions du Genie Civil*, Université de Liège, No. 81, Feb. 1951

Reviewed by Phillip L. Melville

Theoretical equations are derived for the design of concrete mixes by absolute volumes. Feret's and Bolomey's laws are used. A discussion of various aggregate sizes stresses the advantage of gap-gradation. It is indicated that high compaction and low amount of voids in the fresh concrete are now more critical than high cement ratios to obtain high strength concrete. Nine references are addended.

An engineer develops skyhooks for industrial buildings

Cleveland Engineering, V. 44, No. 36, Sept. 6, 1951, pp. 12 and 14

Nontechnical description of an adaptation of hipped plate construction in a small industrial building. The roof deck is a self-load carrying structure supported on walls or beams and columns. The roof valley is supported at either end by a column but no structural beams are required along it since the ribbed slabs act as inclined deep girders. Tie rods prevent spreading of the roof. For longer spans, the roof has a polygonal contour.

Contribution to the calculation of foundations of tall chimneys (Ein Beitrag zur Berechnung von Fundamenten hoher Schornsteine)

A. Mehmel, *Der Bauingenieur* (Berlin), V. 26, No. 10, Oct. 1951, pp. 293-297

Reviewed by Aron L. Mirsky

Analytical study of chimney bases. The author divides them into three elements (1) the outer circular slab, of variable thickness; (2) the circular ring forming the base proper; and (3) the slab of constant thickness inside the ring. As a variation, the case wherein element (2) is so formed that element (3) vanishes is also presented. Formulas for analysis are derived and the results of numerical calculations are given.

Donau bridge at the Ganstor in Ulm (Die Donaubrucke beim Ganstor in Ulm)

Ulrich Finsterwalder and Hermann Konig, *Der Bauingenieur* (Berlin), V. 26, No. 10, Oct. 1951, pp. 289-293

Reviewed by Aron L. Mirsky

Description and discussion of this hingeless, very slender, prestressed ribbed rigid frame, "whose crown and springing are so elastic that the thrust goes through the kern of the crown section and the midpoint of the foundation joint." The bridge consists of two parallel structures, each having two ribs; the span is 82.4 m (270.3 ft) and the over-all height about 15 m (49 ft). The article includes a description of the prestressing and the apparatus used to measure shrinkage and creep in the finished structure.

Method of petrographic examination of aggregates for concrete

Katharine Mather and Bryant Mather, ASTM *Proceedings*, V. 50, 1950, pp. 1288-1313

Authors' Summary

A detailed description of the method for petrographic examination of aggregates for concrete as developed for use by the Concrete Laboratory of the Corps of Engineers. The differences in procedure which depend on the nature of the sample and the purposes of the examination are described. Suggestions are made concerning the features to be delineated, the amount of material to be examined, and the organization and presentation of the results of the examination. One aim of the paper is to clarify the concept of what is meant by petrographic examination of aggregates and, perhaps, ultimately, contribute to greater uniformity in making such examinations.

Changes in American cement specifications (Evolution des normes Americaines des ciments)

J. Cleret de Langavant, *Revue des Materiaux de Construction* (Paris), No. 431-432, Aug.-Sept. 1951, pp. 269-272

Reviewed by Phillip L. Melville

While French standards are subject to very few revisions, American practice calls for frequent changes. Most of the revisions in ASTM standards on cement were on listing of allowed "grinding aids," development of air-entraining agents, regulations on chemical composition of sulfate resisting cements, creation of standards for slag cements, and changes in measurement of fineness. French standards are still more strict than the American on additives to raw cements.

Reinforced concrete—A manual of standard practice

Concrete Reinforcing Steel Institute, Chicago, 1951. 98 pp.

The new edition of this manual, free upon request to engineers and architects in continental United States, describes and illustrates A 305 reinforcing bars available from members of CRSI and gives pertinent ASTM specifications relating to the new type reinforcing bars. Other sections contain data on standard details of design; general specifications for materials, mixing and placing of concrete; standard sizes of bars, spiral rods and column spirals; allowable unit stresses, properties of sections, building code requirements and weights of materials; concrete joist construction; and load table for solid concrete slabs. A worthwhile addition to the engineering library.

Experiences with nondestructive testing of concrete with ball-test apparatus (Erfahrungen uber die zerstorungsfreie Betonprufung mit Kugelprufgeraten)

Hermann Hoeffgen and Georg Back, *Der Bauingenieur* (Berlin), V. 26, No. 10, Oct. 1951, pp. 297-300

Reviewed by Aron L. Mirsky

The authors discuss the effect of various factors—aggregate size, condition and dampness of the test surface, age of concrete, type of apparatus, etc.,—on the relation between the results of the ball-impression tests (similar to the scleroscope hardness tests of metals) and compressive strengths, and conclude that the method is not universally applicable. It will not yield certain quantitative results if the mix was very lean or dry, or the concrete is honeycombed (insufficiently compacted), frozen, too strong or weak (over 350, under 60 kg per cu cm), or where the effect of surface dampness cannot be evaluated.

Latest developments in the design and construction of prestressed concrete sleepers (Derniers progres realises dans le domaine de la traverse en beton precontraint)

Georges Worontzoff, *Communication* No. B 25, International Congress on Prestressed Concrete, Ghent, Belgium, Sept. 1951

Describes developments during the last 10 years in prestressed concrete railroad ties. Major difficulty in design has been the lack of data on the actual forces resulting from traffic and track conditions. Stress calculations are uncertain and the author recommends actual tests in the track. The most important of the stresses are those which result in the section under the rail under wheel load and immediately after passage of the wheel, and those due to the bending and torsion of the middle section.

Either long bench or individual mold systems are used in manufacturing the prestressed ties. An individual mold process of mass production is described.

Final foundation treatment at Hoover Dam

A. Warren Simonds, *Proceedings*, ASCE, V. 77, 1951, Separate No. 109, 22 pp. $0.50

Author's Summary

In the design of Hoover Dam, the Bureau of Reclamation found it necessary to give careful consideration to certain features not subject to an exact mathematical analysis. The treatment of the foundation was one of those matters that involved a design based largely on experience and precedent. Because of the height of the structure and the resulting high head, there was no established antecedent that would furnish a suitable comparison for the foundation treatment. However, pressure grouting was anticipated to reduce seepage, to eliminate uplift pressure in the foundation beneath the dam and appurtenant structures, and to correct defects in the bedrock. In addition to grouting, a drainage system downstream from the grouted area was considered essential. This paper describes the problems encountered in designing the treatment of the foundation, the conditions that developed after the reservoir had filled, and the corrective measures to make foundation conditions agree with the original design assumptions.

Design of arch dams compared with the results of structural model tests (Berechnungen von Bogenstaumauern im Vergleich mit den Ergebnissen statischer Modellversuche)

E. Tschech and F. Jaburek, *Oesterreichische Bauzeitschrift* (Vienna), V. 6, No. 2, Feb. 1951, pp. 17-23
Reviewed by Rudolph Fischl

In connection with the erection of the Salza and Hierzmann dams in Styria, model tests were made at the Technical University of Graz to compare the results with that of a simplified design method.

This method, developed by Professor F. Tölke, is an approximation based on the theory of shells. The author tried to modify the method in some points where differences with the model tests were obvious, as at the restrained parts at the springings. This method is said to be much faster than the trial load method, though it is necessary to revert to the latter to compute the final exact stresses. However, Tölke's method will give quick informations about the properties of a chosen dam section.

Exploratory tests to develop a method for determining the air content of hardened concrete

Alexander Klein, David Pirtz and Milos Polivka, ASTM *Proceedings*, V. 50, 1950, pp. 1283-1286
Authors' Summary

Describes two methods for determining the air content of hardened concrete; (1) an absolute-volume method, which consists in pulverizing the concrete to a given fineness and then, making allowance for evaporable water and impermeable voids, computing the air content from the difference between the over-all volume of the concrete specimen and its absolute volume after pulverizing; and (2) a vacuum-saturation method in which the air content is computed from the change in weight of a specimen due to saturation. The saturation of a standard-cured specimen is accomplished by first subjecting the specimen to vacuum and then saturating it with water under pressure.

Air contents as obtained by the two methods are compared to the air content of the corresponding fresh concrete as determined by ASTM C 173-42 T. The absolute-volume method gives lower values of air content than the vacuum-saturation method. Both methods appear to be of equal reproducibility. The vacuum-saturation method is simpler and more rapid. Results show that the air content of concrete decreases with age and that the type of agent used has an effect on this reduction in air content.

Cementing agents for mass concrete: Investigations of hydraulic cementing agents of cement, lime and trass (Bindemittel fur Massenbeton: Untersuchungen uber hydraulische Bindemittel aus Zement, Kalk und Trass)

Kurt Walz, German Committee for Reinforced Concrete (Deutscher Ausschuss für Stahlbeton), No. 104, 1951, Wilhelm Ernst & Sohn, Berlin. 68 pp. 5.50 DM.
Reviewed by Aron L. Mirsky

Report of investigations, conducted from 1941 to 1949, on various aspects of cement-lime-trass cementing agents for mass concrete: characteristics, flexural and compressive strengths, watertightness, heat development, and durability (against frost, weather, and chemically aggressive water). The author concludes that 250 kg per cu m of concrete of his Type I (35 percent portland cement, 10 percent lime, 55 percent trass) is better than portland cement alone for watertight concrete, but that for weather permanence each cu m of concrete should contain about 300 kg of Type I or 250 kg of his Type II (45 percent portland cement, 10 percent lime, 45 percent trass). Type I is, however, better in sulfate water than Type II.

New shell constructions using Zeiss-Dywidag system (Neue Schalenbauten System Zeiss-Dywidag)

Hans-Joachim Eulitz, *Beton- u. Stahlbetonbau* (Berlin), V. 46, No. 1, Jan. 1951, pp. 1-5
Reviewed by Rudolph Fischl

Since the development of shell construction by Carl Zeiss and Dyckerhoff & Widmann Co. in 1923, more than 20 million sq ft of roof shells have been built, thus proving its variety and economy. Due to the shortage of wood at the end of the war, precast shell constructions have been developed and successfully erected. The paper describes two market halls and four industrial plants, all built after 1945, where shell roofs were used, with spans of the vaults up to 82 ft 6 in. longitudinally and 46 ft transversally. Very low maintenance costs, excellent lighting conditions, high fire resistance and other advantages are the reasons that this construction has been more and more preferred for industrial buildings. Illustrations showing the interior halls give an impression of the unusual open effect of this type of construction.